Statistical Modelling of Marine Turbulence

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- Mixing vs. stratification
- General Ocean turbulence Model (GOTM)
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- General Estuarine Transport Model (GETM)
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Basic approaches

Two approaches to averaged turbulence modelling:

- **Statistical turbulence modelling:** Convert NSE to Friedmann-Keller series, cut-off where suitable and parameterise unknown terms.
- **Empirical turbulence modelling:** Close equations on lowest order and parameterise relevant processes.

NSE → Reynolds equation

Momentum Equation:

$$\partial_t v_i + v_j \partial_j v_i - \nu \partial_{jj} v_i + 2\varepsilon_{ijl}\Omega_j v_l = -\frac{\partial_i p}{\rho_0} - \frac{g_i}{\rho_0}\rho.$$

Reynolds averaging:

$$v_i = \bar{v}_i + \tilde{v}_i, \quad \rho = \bar{\rho} + \tilde{\rho}, \ldots$$

Reynolds Equation:

$$\partial_t \bar{v}_i + \bar{v}_j \partial_j \bar{v}_i - \partial_j \left(\nu \partial_j \bar{v}_i - \langle \tilde{v}_j \tilde{v}_i \rangle \right) + 2\varepsilon_{ijl}\Omega_j \bar{v}_l = -\frac{\partial_i \bar{p}}{\rho_0} - g_i \frac{\bar{\rho}}{\rho_0}.$$

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Reynolds Stress Equation

$$
\begin{aligned}
\partial_t \langle \tilde{v}_i \tilde{v}_j \rangle + \partial_l \left(\bar{v}_l \langle \tilde{v}_i \tilde{v}_j \rangle + \langle \tilde{v}_l \tilde{v}_i \tilde{v}_j \rangle - \nu \partial_l \langle \tilde{v}_i \tilde{v}_j \rangle \right) \\
= \underbrace{-\partial_l \bar{v}_i \langle \tilde{v}_l \tilde{v}_j \rangle - \partial_l \bar{v}_j \langle \tilde{v}_l \tilde{v}_i \rangle}_{P_{ij}} \\
- \underbrace{2\Omega_l (\varepsilon_{ilm} \langle \tilde{v}_j \tilde{v}_m \rangle + \varepsilon_{jlm} \langle \tilde{v}_i \tilde{v}_m \rangle)}_{\Omega_{ij}} \\
- \underbrace{\frac{1}{\rho_0} \{ g_i \langle \tilde{v}_j \tilde{\rho} \rangle + g_j \langle \tilde{v}_i \tilde{\rho} \rangle \}}_{B_{ij}} - \underbrace{\frac{1}{\rho_0} (\langle \tilde{v}_i \partial_j \tilde{p} + \tilde{v}_j \partial_i \tilde{p} \rangle)}_{\Pi_{ij}} \\
- \underbrace{2\nu \langle (\partial_l \tilde{v}_j)(\partial_l \tilde{v}_i) \rangle}_{\varepsilon_{ij}} .
\end{aligned}
$$

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Algebraic SMCs

The following steps lead to different types of second-moment closures:

- Empirical closures of pressure-strain correlators.
- Neglect or simplification of advective and diffusive fluxes of second-moments.
- Neglect of rotational terms in the second-moment equations.
- Boundary layer assumption (neglect of horizontal gradients and non-hydrostatic effects).
- ... and many more details ...

Algebraic SMCs

Turbulent Fluxes:

$$\langle \tilde{u}\tilde{w} \rangle = -\nu_t \partial_z \bar{u} , \quad \langle \tilde{w}\tilde{T} \rangle = -\nu_t' \partial_z \bar{T}$$

Eddy Viscosity / Eddy Diffusivity:

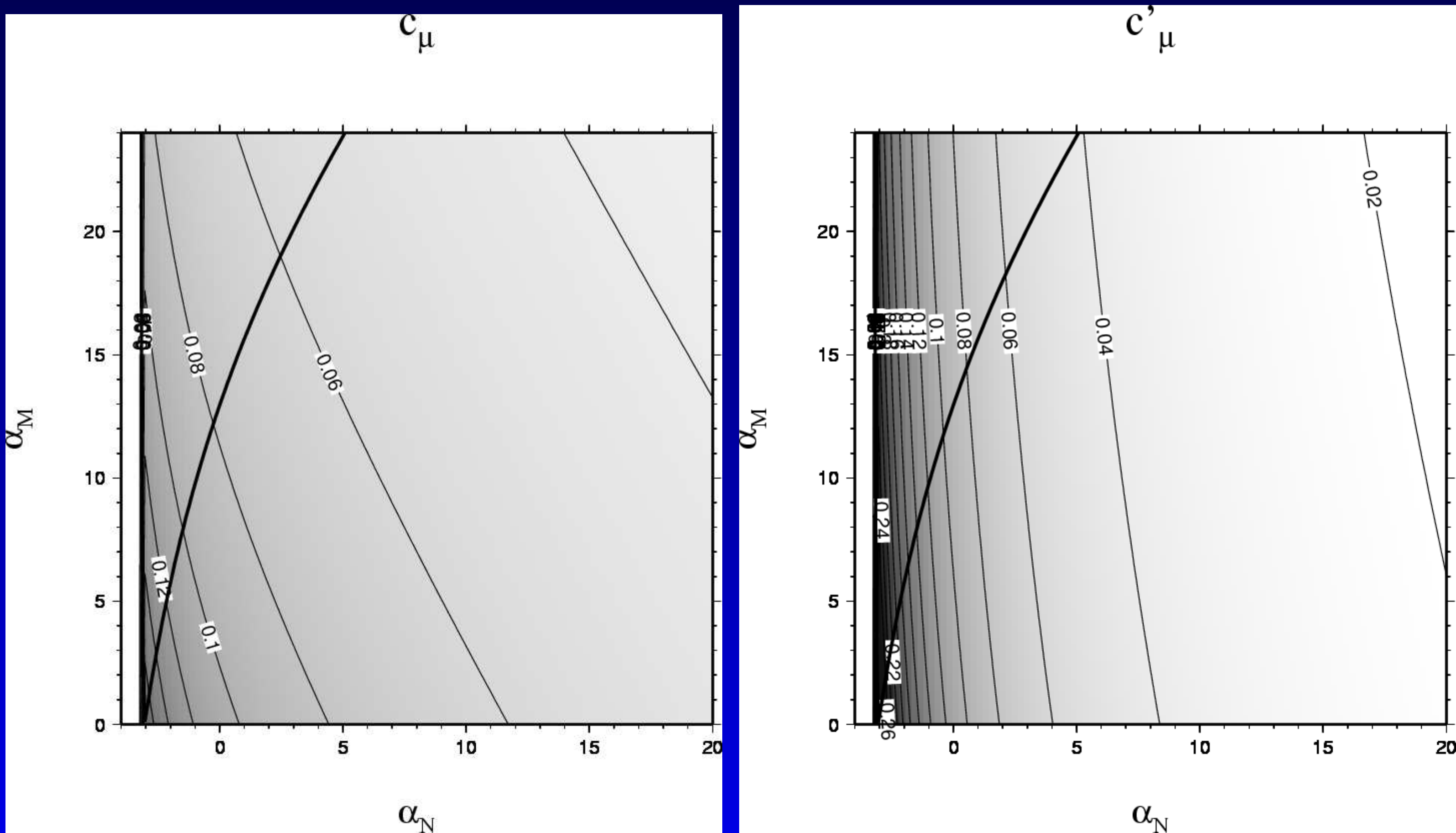
$$\nu_t = c_\mu(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon} , \quad \nu_t' = c_\mu'(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon} .$$

Shear Number, Buoyancy Number:

$$\alpha_M = \frac{k^2}{\varepsilon^2} M^2 , \quad \alpha_N = \frac{k^2}{\varepsilon^2} N^2 .$$

Stability Functions

Canuto et al. [2001]:



Exact TKE-Equation

$$\partial_t k + \partial_j \left(\bar{v}_j k + \langle \tilde{v}_j \frac{1}{2} \tilde{v}_i^2 \rangle - \nu \partial_j k + \frac{1}{\rho_0} \langle \tilde{v}_j \tilde{p} \rangle \right)$$

$$= \underbrace{-\langle \tilde{v}_j \tilde{v}_i \rangle \partial_i \bar{v}_j}_{P} \underbrace{-\frac{g}{\rho_0} \langle \tilde{v}_3 \tilde{\rho} \rangle}_{B} - \underbrace{\nu \langle (\partial_j \tilde{v}_i)^2 \rangle}_{\varepsilon},$$

This TKE equation will be modelled as it is given above, the only parameterisations needed are for the turbulent flux terms, for which usually the down-gradient approximation is used.

Dissipation equation

Exact form (e.g. *Wilcox* [1998]):

$$
\begin{aligned}
&\partial_t \varepsilon + \partial_j \left(\bar{v}_j \varepsilon + \langle \tilde{v}_j \nu (\partial_j \tilde{v}_i)^2 \rangle - \nu \partial_j \varepsilon + 2 \tfrac{\nu}{\rho_0} \langle \partial_i \tilde{v}_j \partial_i \tilde{p} \rangle \right) \\
&\quad = \underbrace{-2\nu \partial_j \bar{v}_i \left(\langle \partial_i \tilde{v}_k \partial_j \tilde{v}_k \rangle + \langle \partial_k \tilde{v}_i \partial_k \tilde{v}_j \rangle \right) - 2\nu \partial_{jk} \bar{v}_i \langle \tilde{v}_k \partial_j \tilde{v}_i \rangle}_{P_\varepsilon} \\
&\qquad \underbrace{-2\nu \tfrac{g}{\rho_0} \partial_j \langle \tilde{v}_3 \partial_j \tilde{\rho} \rangle}_{B_\varepsilon} - \underbrace{2\nu \left(\langle \partial_j \tilde{v}_i \partial_k \tilde{v}_i \partial_j \tilde{v}_k \rangle + \nu \langle (\partial_{ij} \tilde{v}_k)^2 \rangle \right)}_{\varepsilon_\varepsilon}
\end{aligned}
\tag{1}
$$

k-ε model (*Launder and Spalding* [1972]):

$$
\partial_t \varepsilon - \partial_z \left(\frac{\nu_t}{\sigma_\varepsilon} \partial_z \varepsilon \right) = \frac{\varepsilon}{k} \left(c_{\varepsilon 1} P + c_{\varepsilon 3} B - c_{\varepsilon 2} \varepsilon \right) .
$$

Mellor-Yamada model

General relation between k, ε and L:

$$L = c_\mu^{3/4} \frac{k^{3/2}}{\varepsilon}$$

k-kL model (*Mellor and Yamada* [1982]):

$$\partial_t (kL) - \partial_z (S_l \partial_z (kL)) =$$

$$\frac{L}{2} \left[E_1 P + E_3 B - \left(1 + E_2 \left(\frac{L}{L_z} \right)^2 \right) \varepsilon \right] .$$

Length scale equations (cont'd)

Other approaches are using equations for $\omega = \varepsilon/k$ (k-ω model), k/ε, k^2/ε, ..., so why not using the generalised approach of a $k^n\varepsilon^m$ equation ?

Generic length scale equation (*Umlauf and Burchard* [2003]):

$$\partial_t \left(k^n\varepsilon^m\right) - \partial_z \left(\frac{\nu_t}{\sigma_{nm}} k^n\varepsilon^m\right) =$$

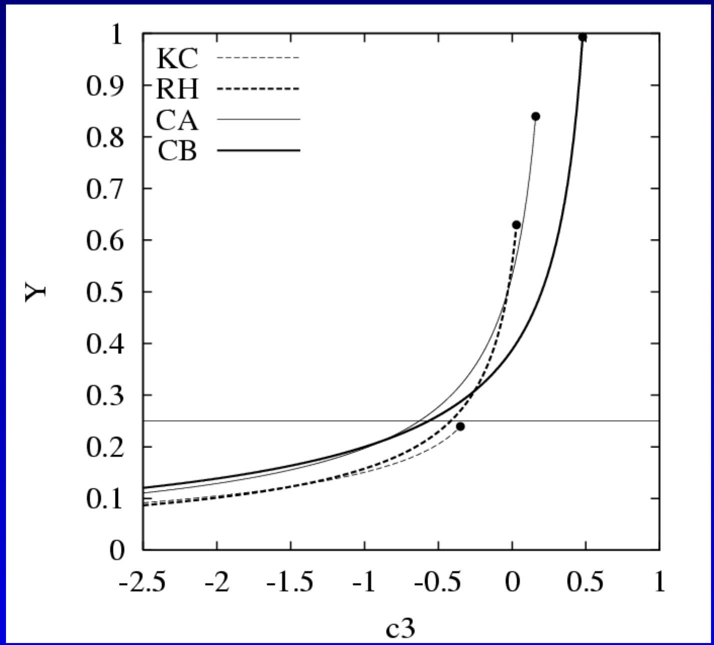
$$k^{n-1}\varepsilon^m \left(c_{nm1}P + c_{nm3}B - c_{nm2}\varepsilon\right)$$

This works without correction term only for $m \geq 0$.

Total equilibrium (k-ε)

$$\dot{k} = \dot{\varepsilon} \Longrightarrow R_i = \frac{-\frac{g}{\rho_0}\partial_z \rho}{(\partial_z u)^2 + (\partial_z v)^2} = R_i^{st} = \frac{c_\mu}{c'_\mu} \cdot \frac{c_{2\varepsilon} - c_{1\varepsilon}}{c_{2\varepsilon} - c_{3\varepsilon}}.$$

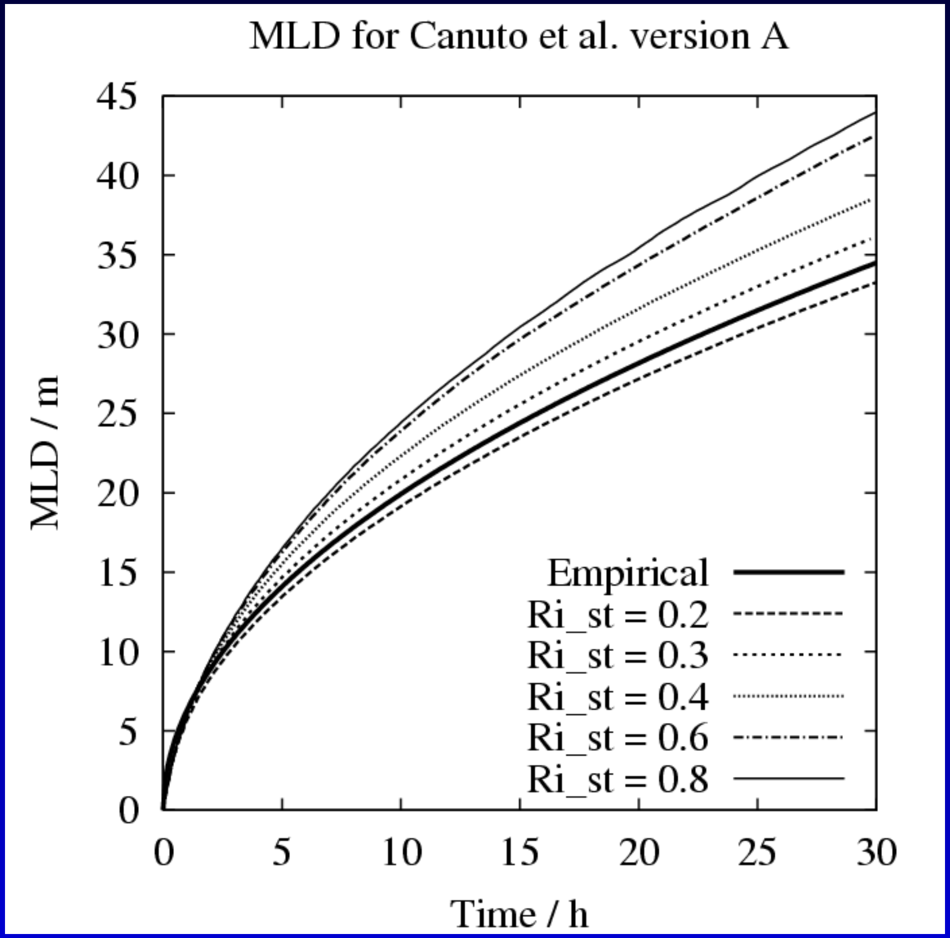
$R_i^{st} \approx 0.25$: Steady-state Richardson number.



Burchard & Bolding [2001]

Kato-Phillips experiment

Wind-induced mixed-layer depth (MLD)



Burchard & Bolding [2001]

GOTM (General Ocean Turbulence Model) - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address http://www.gotm.net

GO TM

- Challenge
  - Aim
  - The Idea
  - Key features
- Software
  - Fortran code
  - Test cases
  - Forcing
  - How to run?
- Information
  - What's New
  - Publications
  - E-mail list
  - FAQ
  - User Group
  - Hot Links
  - Who's Who?

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General Ocean Turbulence Model

GOTM is a one-dimensional numerical model developed and supported by a core team of ocean modellers. GOTM aims at simulating accurately vertical exchange processes in the marine environment where mixing is known to play a key role. GOTM is freely available under the GPL (Gnu Public License).

If you are interested in using GOTM, you can download:

Software: Fortran source

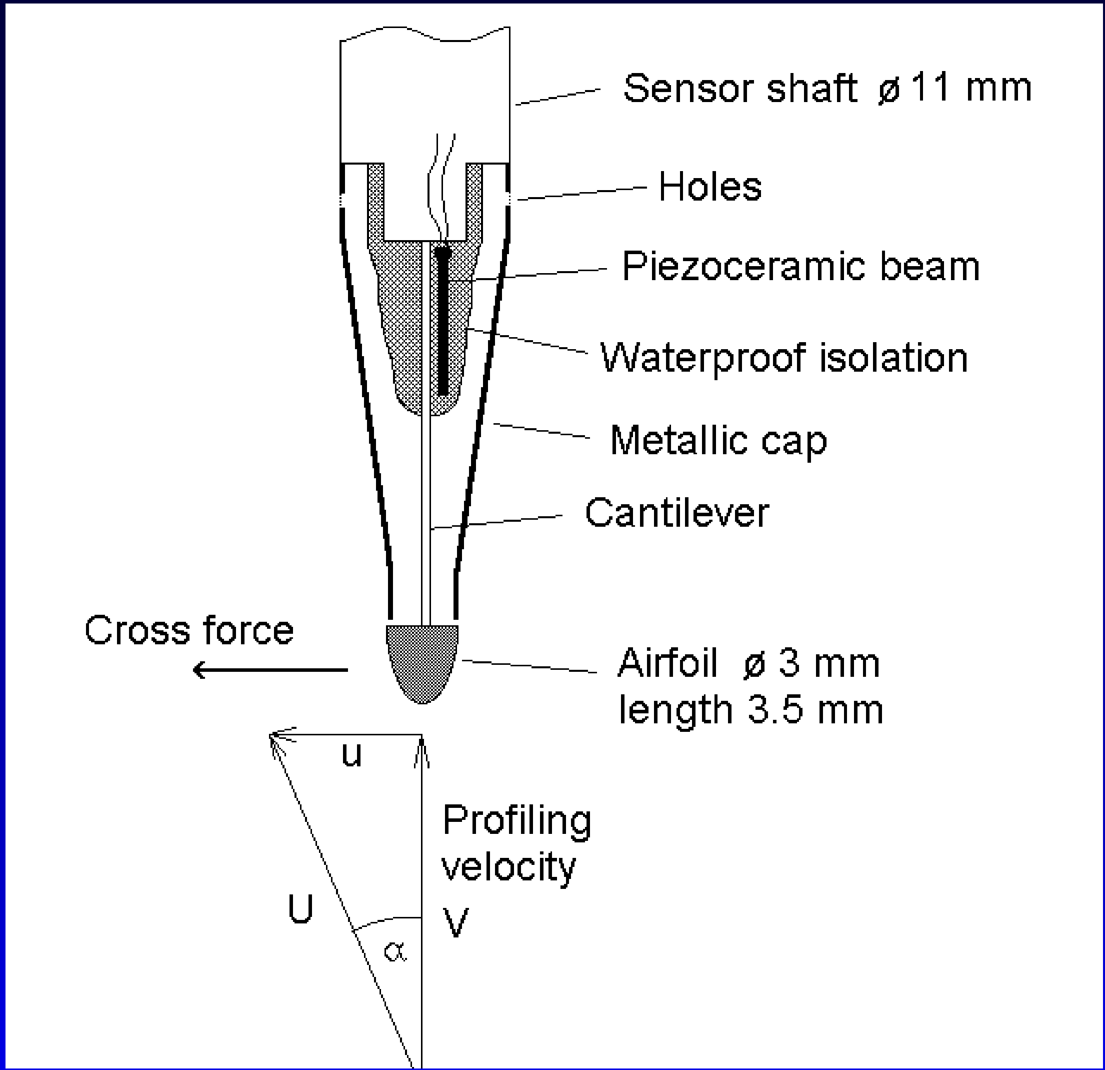
Examples: Test cases - Outputs

Documentation: Summary - 2003 report - 1999 report

You are warmly invited to join the GOTM mailing list and send any comments/questions to the GOTM developers or become a GOTM contributor. The GOTM developers are grateful to their sponsors.

Microstructure observations

Shear probe mounted on free-falling profiler



Pers. comm. Hartmut Prandke

Microstructure observations

Definition of dissipation rate:

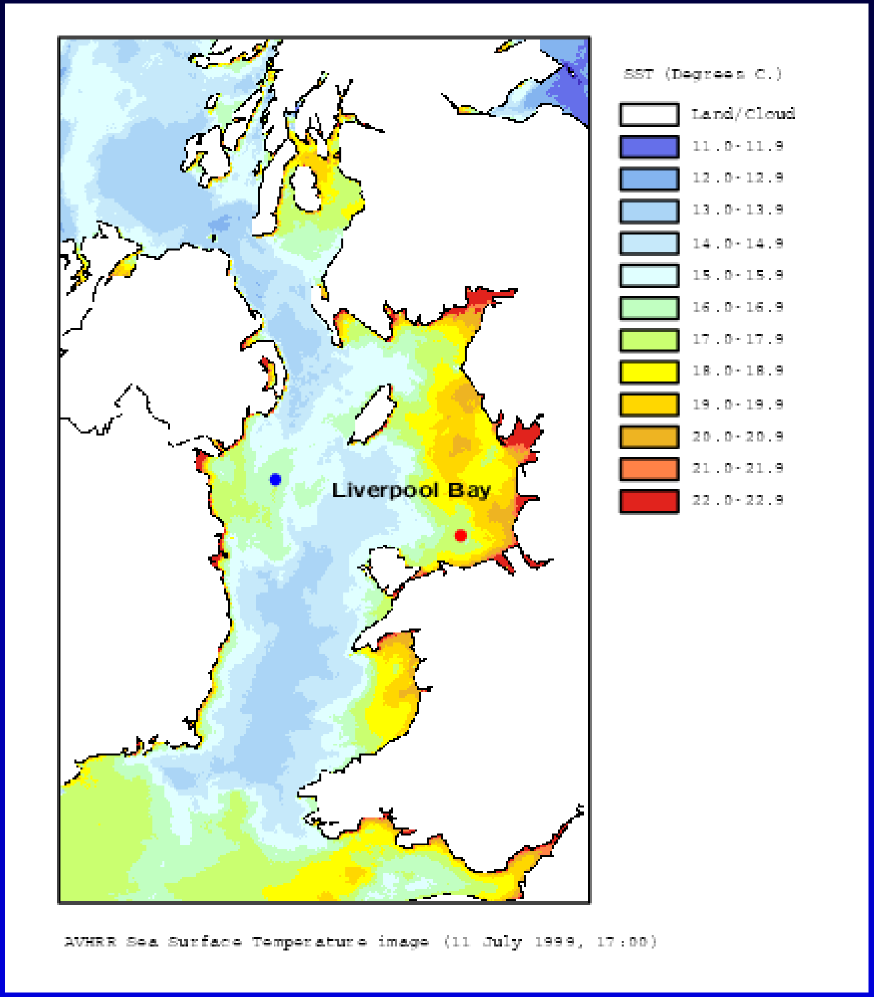
$$\varepsilon = \nu \sum_{i,j=1}^{3} \left\langle \left(\frac{\partial \tilde{v}_i}{\partial x_j} \right)^2 \right\rangle$$

Turbulence isotropic at small scales:

$$\varepsilon = 7.5\,\nu \left\langle \left(\frac{\partial \tilde{u}}{\partial z} \right)^2 \right\rangle$$

Liverpool Bay

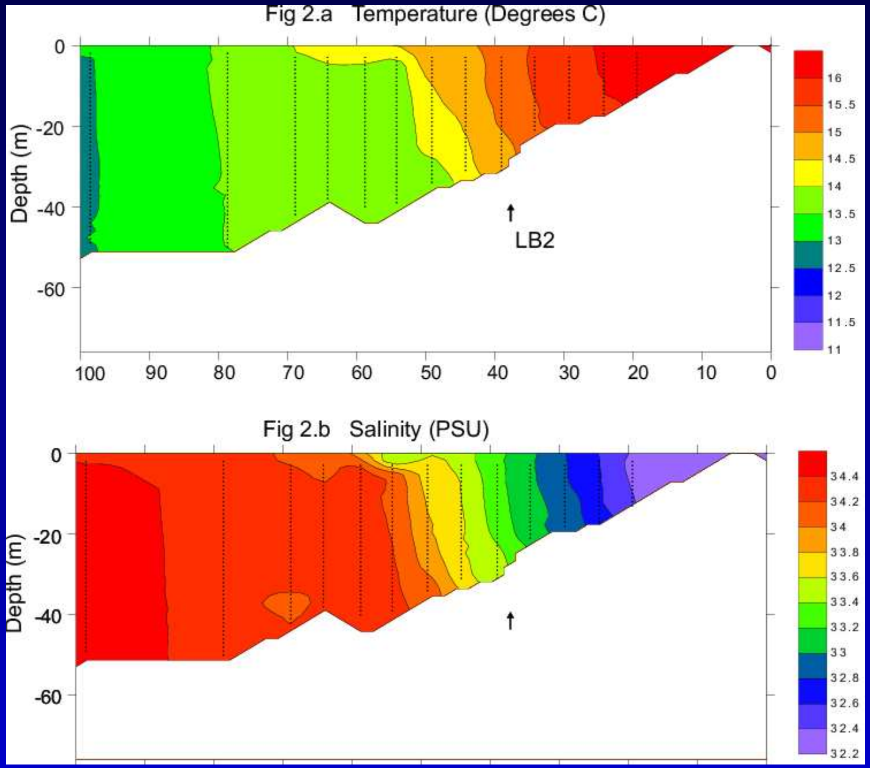
SST from space and location of station (●)



Courtesy to School of Ocean Sciences, UBW, Wales

Liverpool Bay

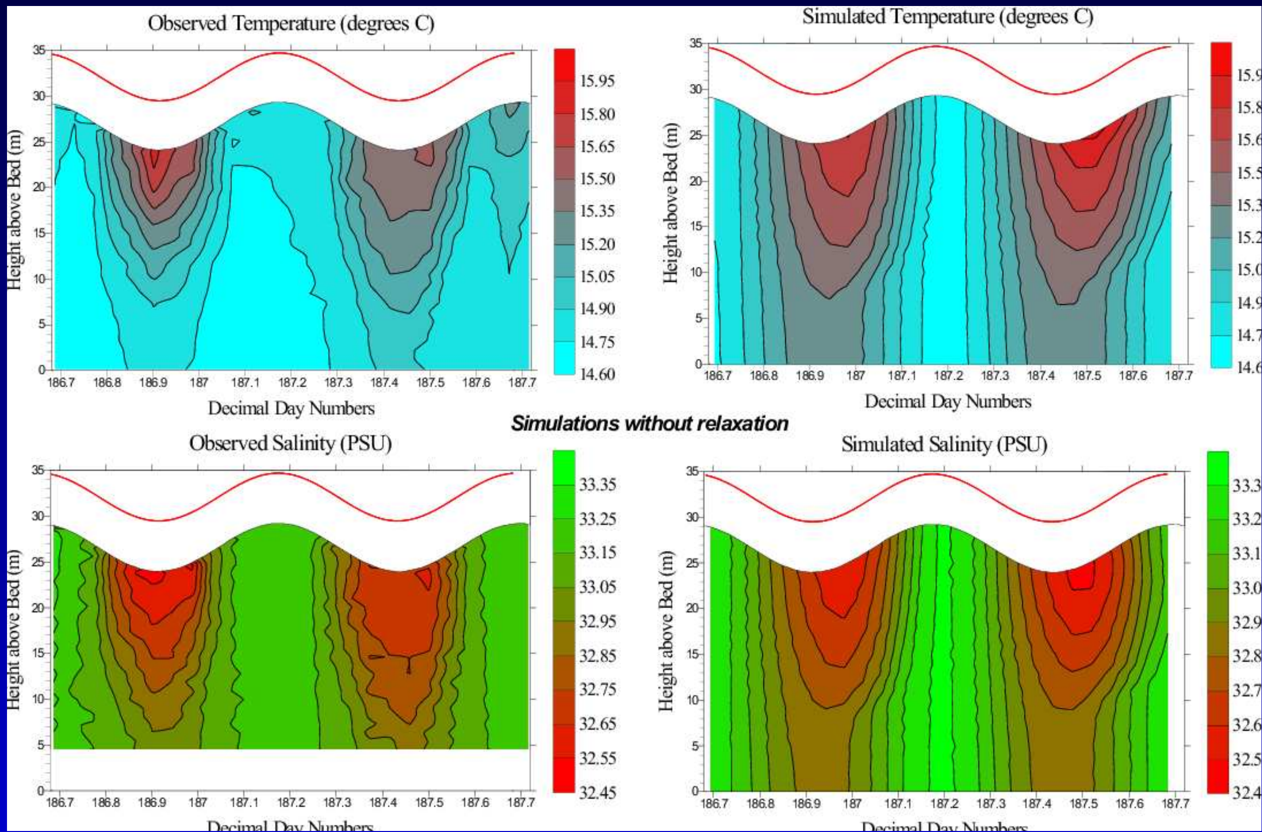
Section of Temperature and Salinity



Rippeth, Fisher, Simpson [2001]

Liverpool Bay

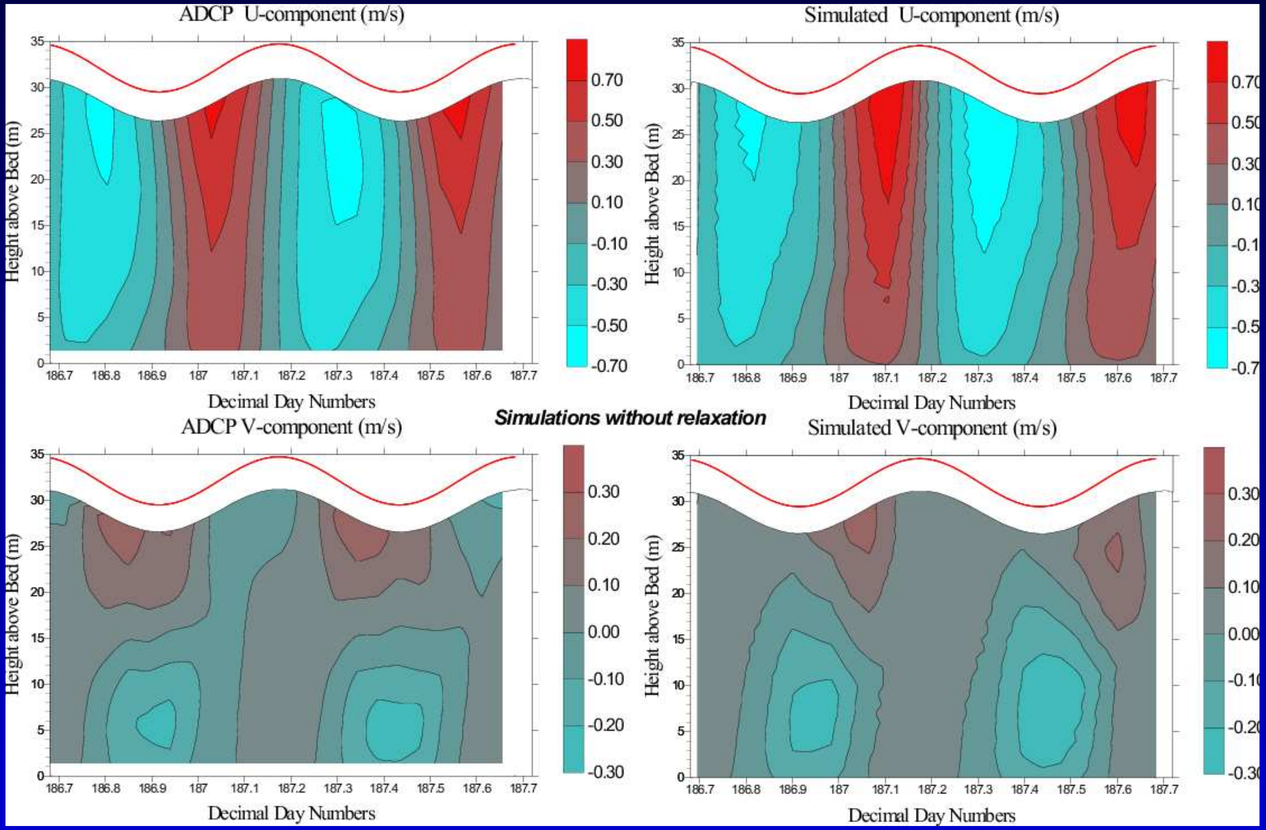
Observed and simulated temperature and salinity



Simpson, Burchard, Fisher, Rippeth [2002]

Liverpool Bay

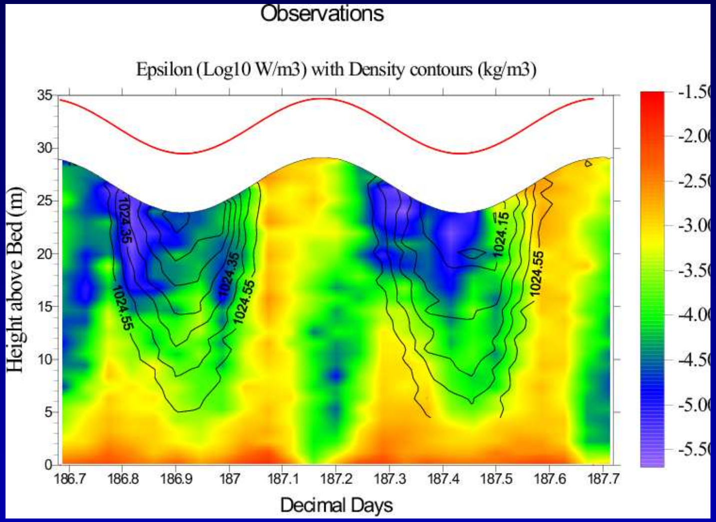
Observed and simulated current velocity

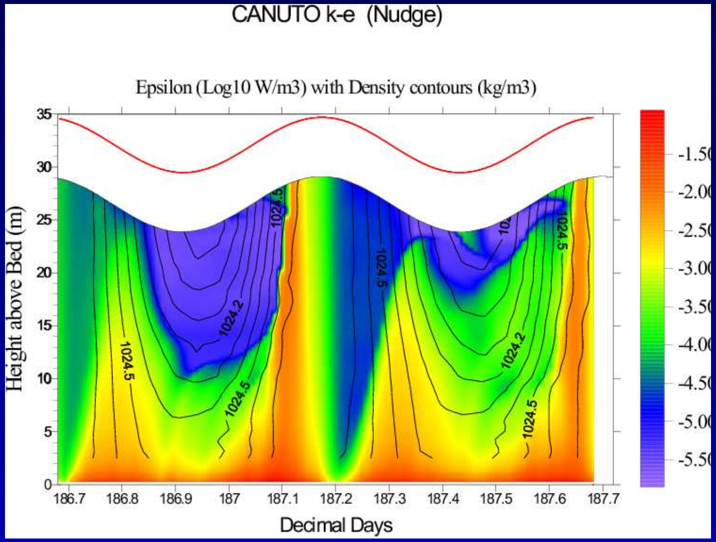


Simpson, Burchard, Fisher, Rippeth [2002]

Liverpool Bay

Observed and simulated dissipation rates

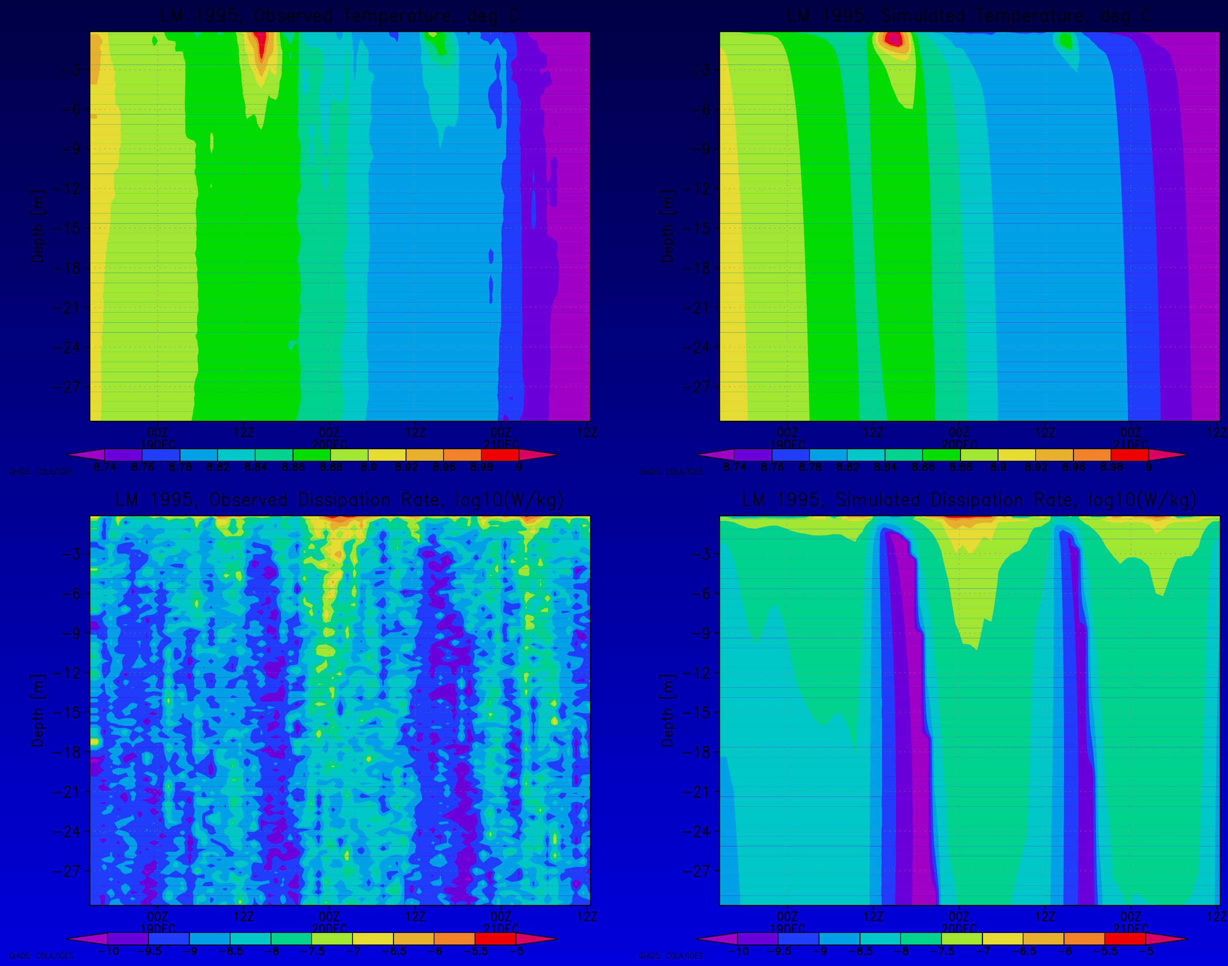




Simpson, Burchard, Fisher, Rippeth [2002]

Lago Maggiore, Italy

Observations and simulations of T and ε (Stips et al. [2002])



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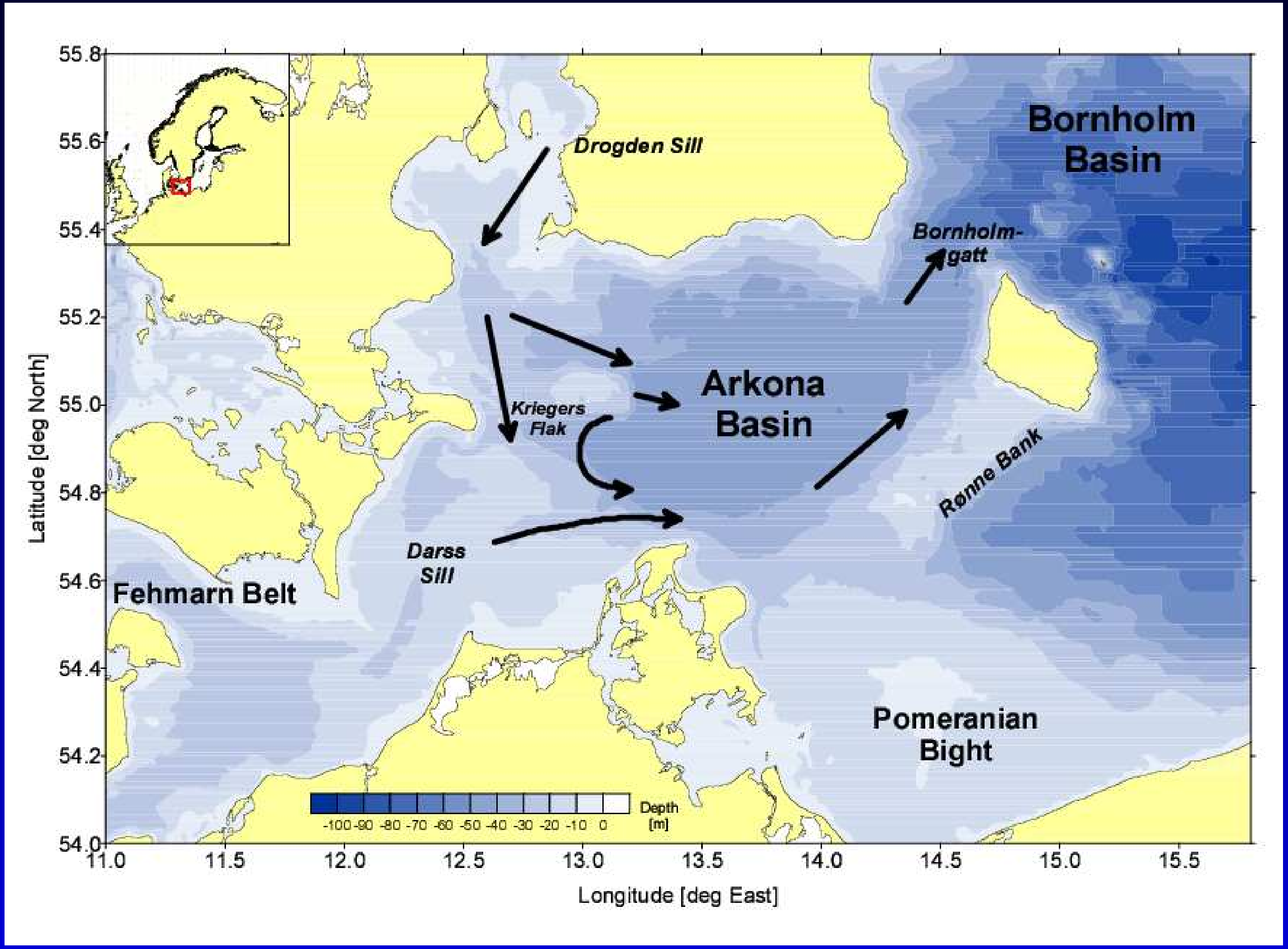
General Estuarine Transport Model

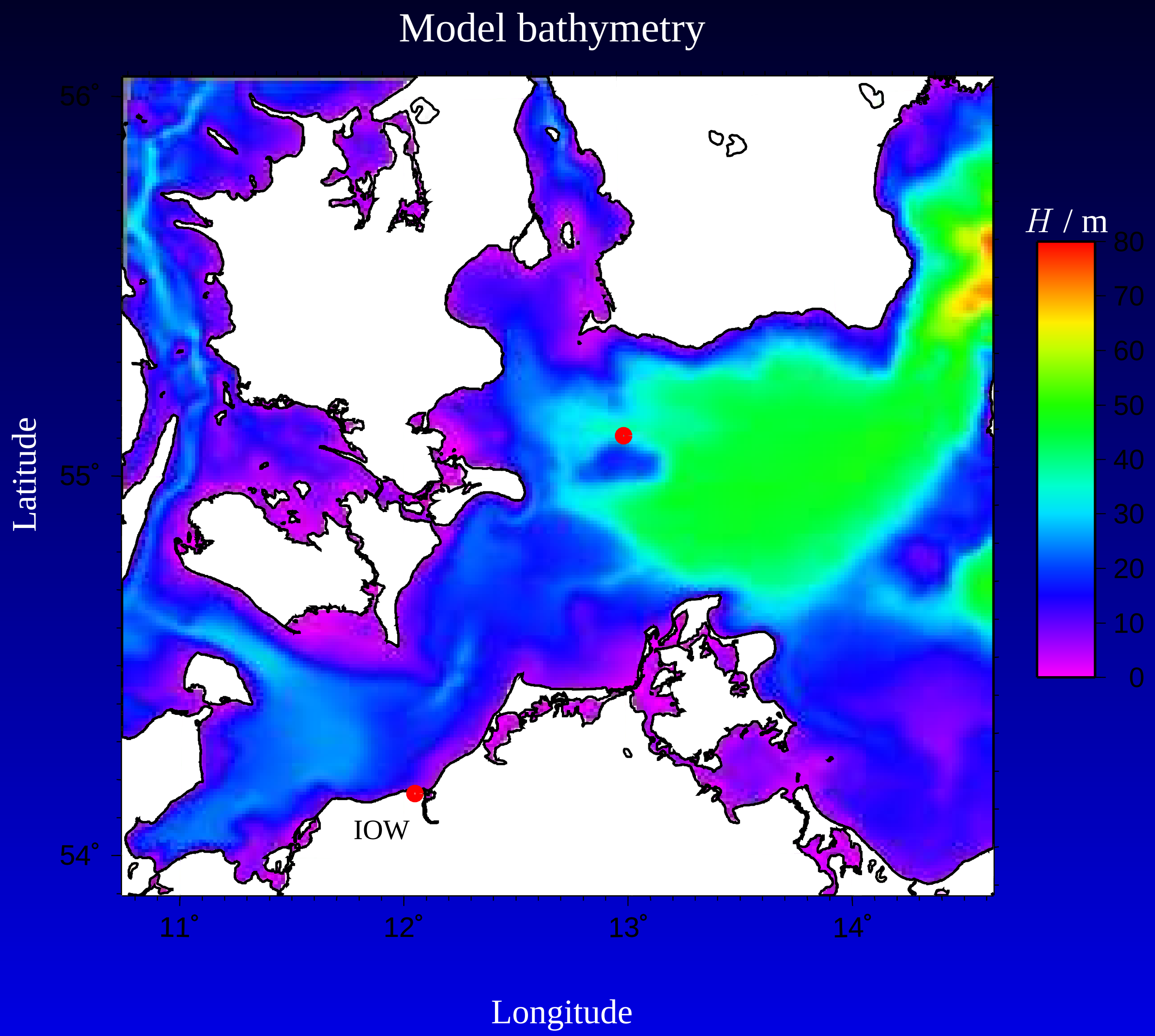
- Three dimensional, hydrostatic, free surface, baroclinic
- Mode-splitting, Arakawa-C grid
- Horizontal coord.: Cartesian, spherical or orthogonal
- Vertical coord.: Sigma, z-levels or generalised
- Turbulence closures from GOTM (`http://www.gotm.net`)
- Various advection schemes for momentum and tracers
- Stable drying and flooding algorithm
- Fully parallelised (domain decomposition)
- Public Domain (`http://www.bolding-burchard.com/getm`)

Arkona Sea



Graphics by Volker Mohrholz, IOW

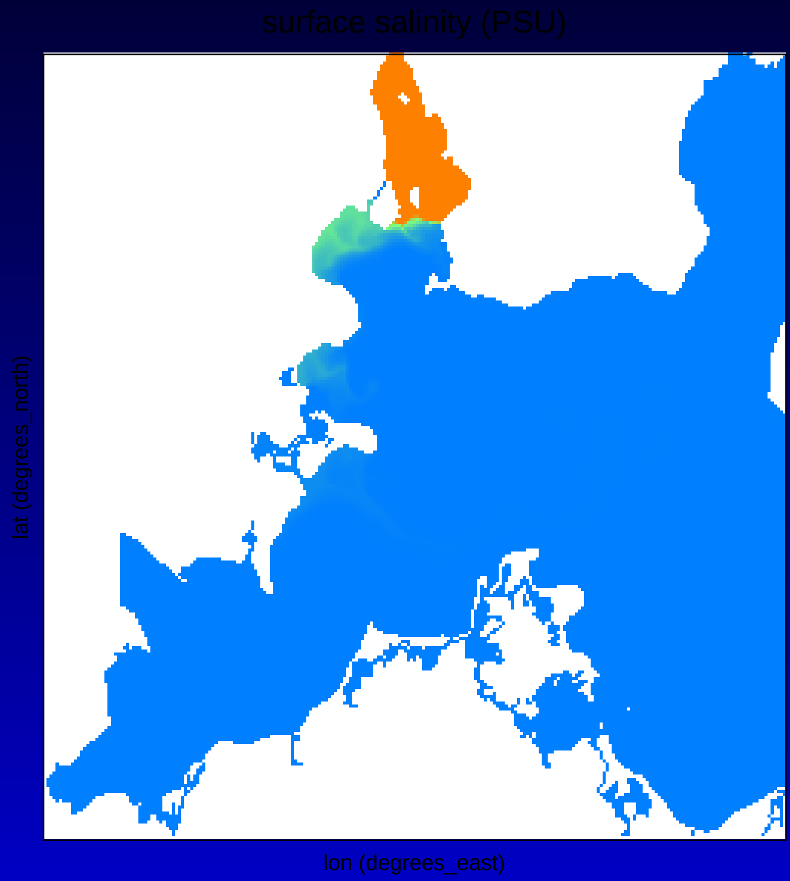
Arkona Sea



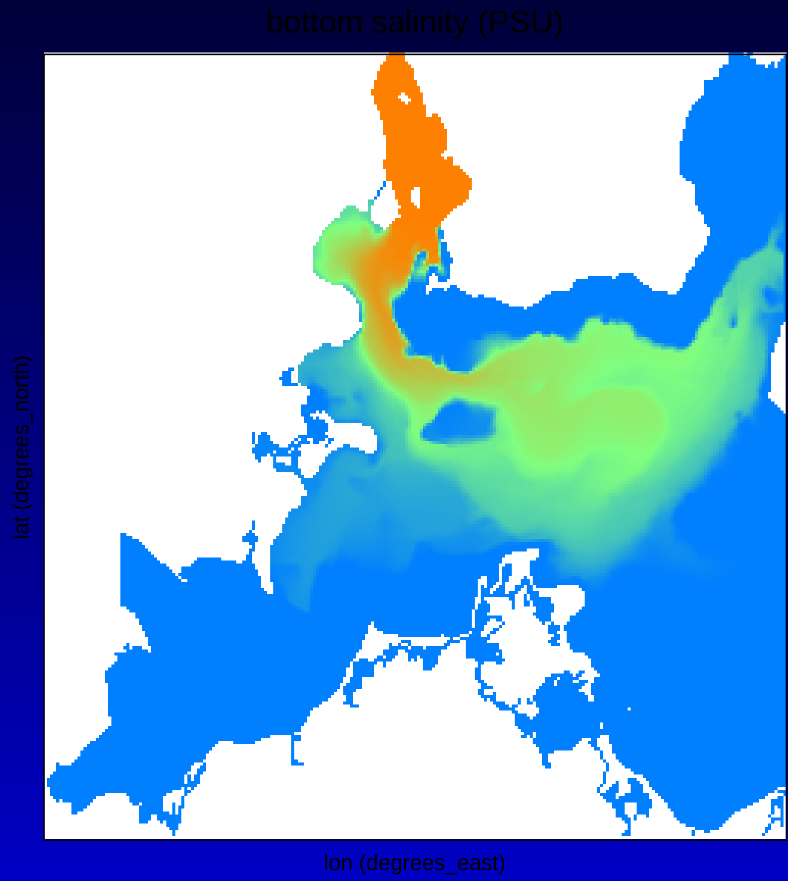
●: QuantAS station

Arkona Sea

Surface salinity (8-25 psu)



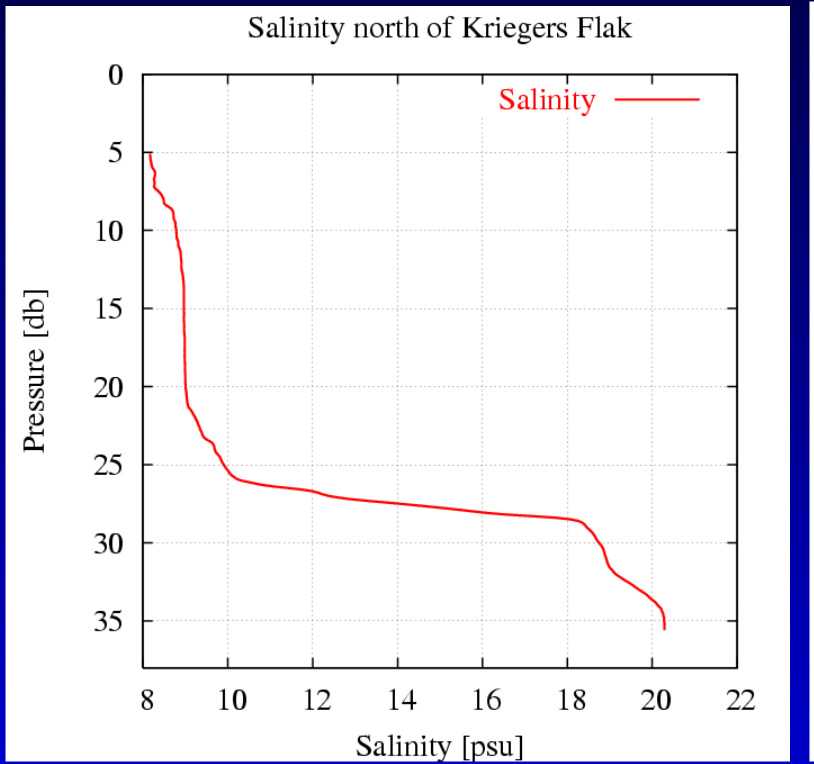
Bottom salinity (8-25 psu)

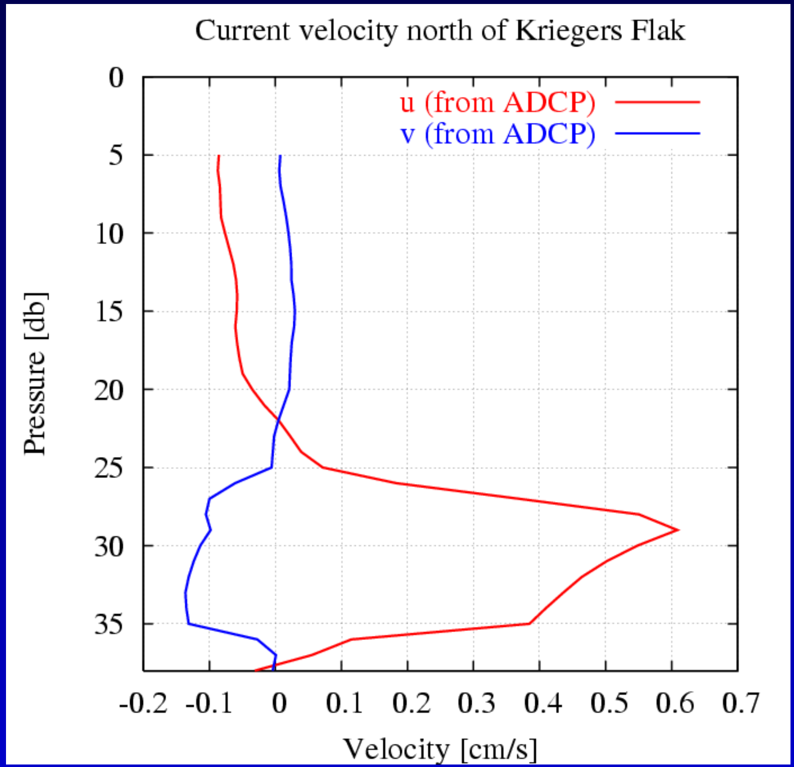


Simulation with GETM

Arkona Sea

Observations from moored ship (MzB Helmsand)

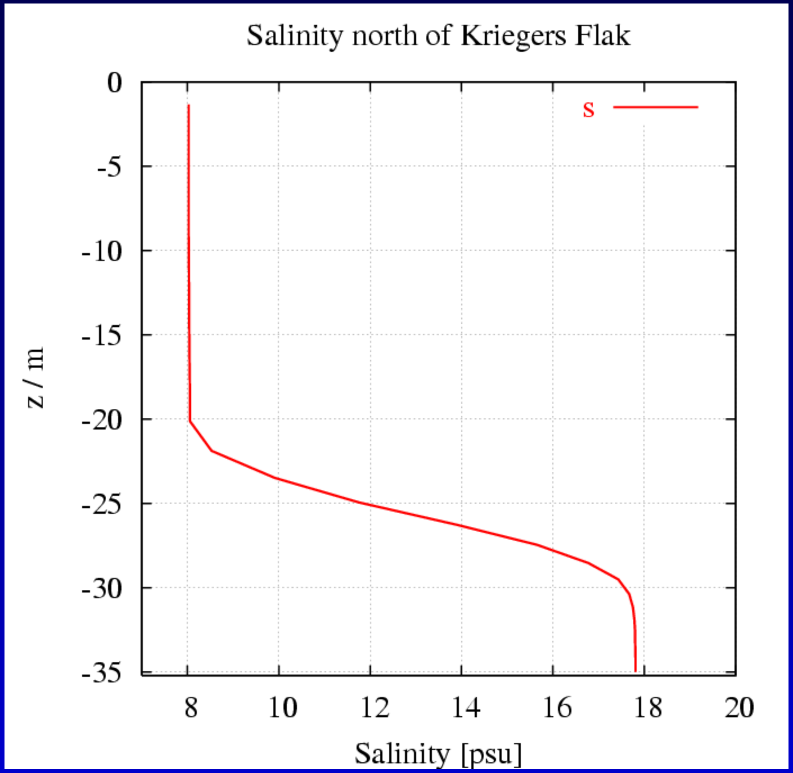


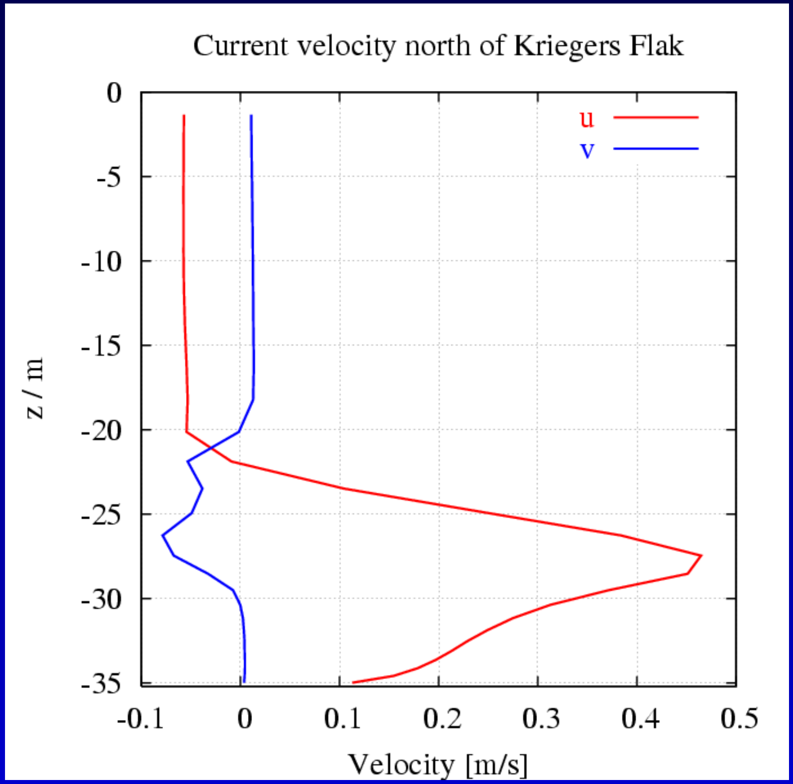


Data by Jürgen Sellschopp, Volker Fiekas, FWG Kiel

Arkona Sea

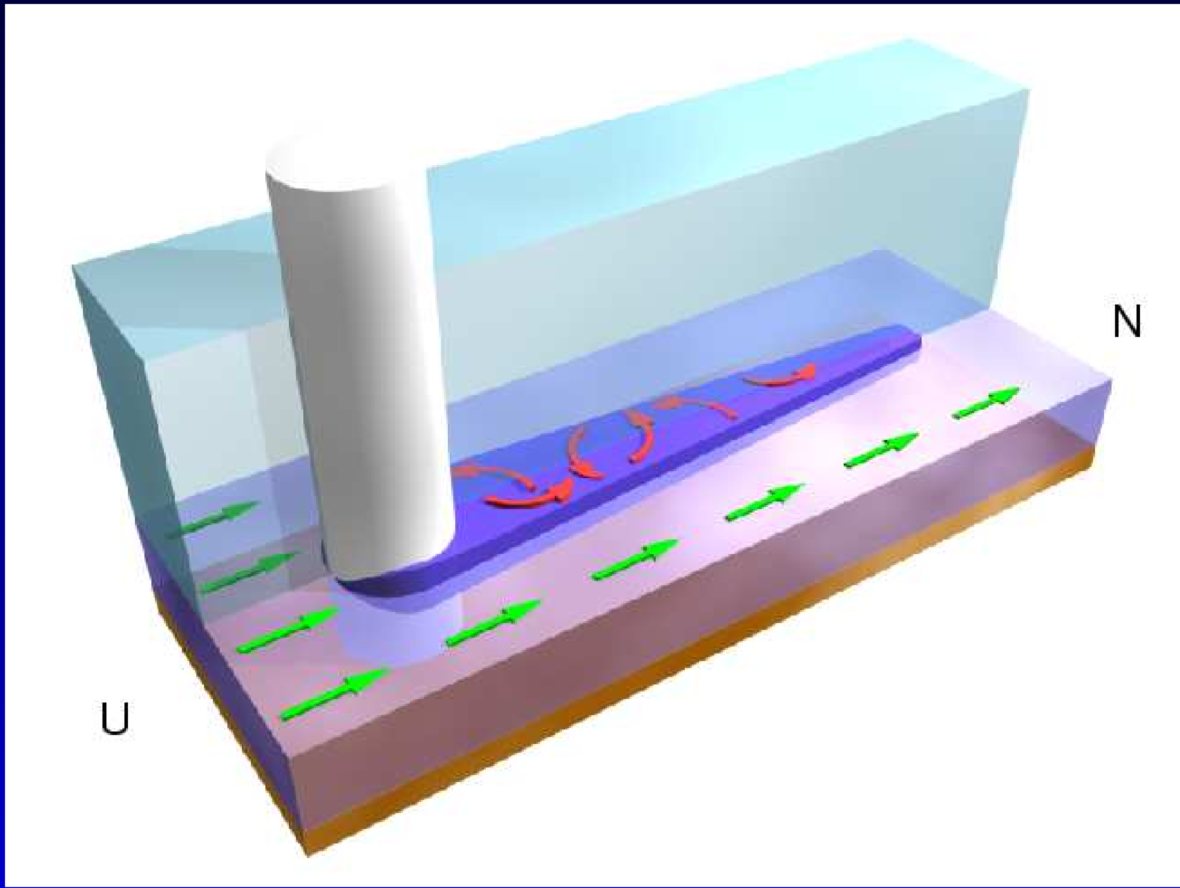
Idealised simulations (with GETM)





Offshore Constructions

Additional mixing by Offshore Wind Farms ?



Graphics by Jan Donath (IOW)

Conclusions

Quantitative numerical simulations of buoyancy effects in the coastal zone require models with certain properties such as

- Higher turbulence closures
- Surface and bottom following coordinates
- Positive definite and monotone advection schemes

Advantageous are highly flexible vertical and horizontal coordinates.

Thanks to ...

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