Adaptive vertical grids in numerical ocean models

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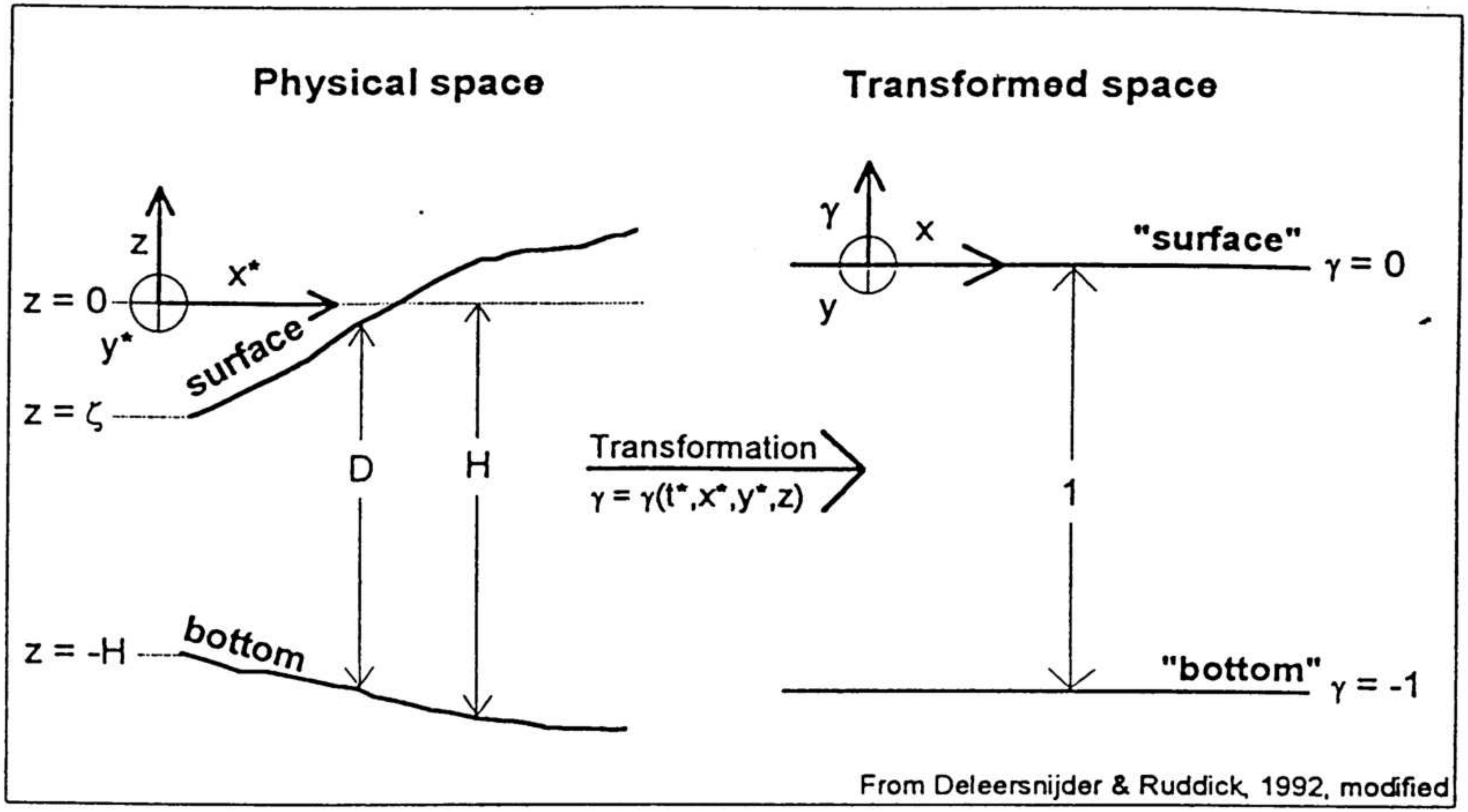
Coordinate transformation



Coordinate transformation

Salinity equation (2D) in Cartesian cooordinates:

$$\partial_t^* S + \partial_x^*(uS) + \partial_z^*(wS) - \partial_z^* \left(\nu_t' \partial_z^* S\right) = 0.$$

Coordinate transformation:

$$\gamma = \gamma\left(t^*, x^*, z\right) \quad \Leftrightarrow \quad z = z\left(t, x, \gamma\right).$$

Transformed vertical velocity, Jacobian:

$$\tilde{\omega} = \partial_t^* \gamma + u\partial_x^* \gamma + w\partial_z^* \gamma, \quad J = \partial_\gamma z = \left(\partial_z^* \gamma\right)^{-1}.$$

Transformed salinity equation:

$$\partial_t(JS) + \partial_x(JuS) + \partial_\gamma(\tilde{\omega}S) - \partial_\gamma \left(\frac{\nu_t'}{J}\partial_\gamma S\right) = 0.$$

Vertical discretisation

Vertical discretisation of γ-space:

$$-1 = \gamma_0 < \gamma_1 < \cdots < \gamma_{N-1} < \gamma_N = 1.$$

Layer integration:

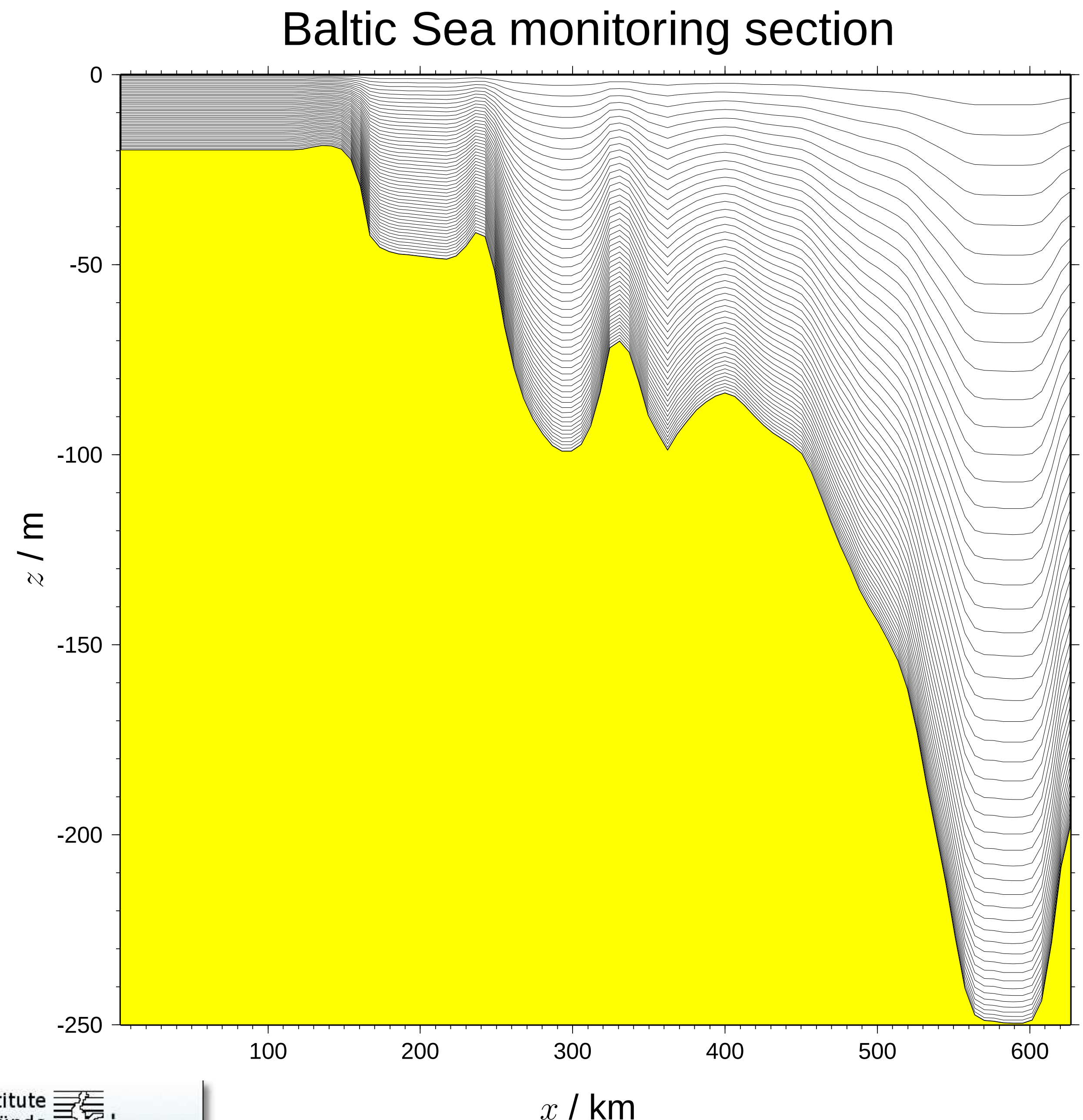
$$h_k = \int_{\gamma_{k-1}}^{\gamma_k} J \, d\gamma; \quad S_k = \frac{1}{h_k} \int_{\gamma_{k-1}}^{\gamma_k} JS \, d\gamma; \quad p_k = \int_{\gamma_{k-1}}^{\gamma_k} Ju \, d\gamma.$$

Layer integrated salinity equation (F_k: turbulent flux):

$$\partial_t \left(h_k S_k\right) + \partial_x \left(p_k S_k\right) + \left(\tilde{\omega} S\right)_k - \left(\tilde{\omega} S\right)_{k-1}$$

$$-F_k + F_{k-1} = 0.$$

Non-adaptive γ-coordinate



Adaptive grids

Diffusion equation for coordinate levels z:

$$\partial_t z - \partial_\sigma \left(k^{grid} \partial_\sigma z \right) = 0, \quad \sigma = \frac{z-\eta}{D}.$$

Diffusion coefficient:

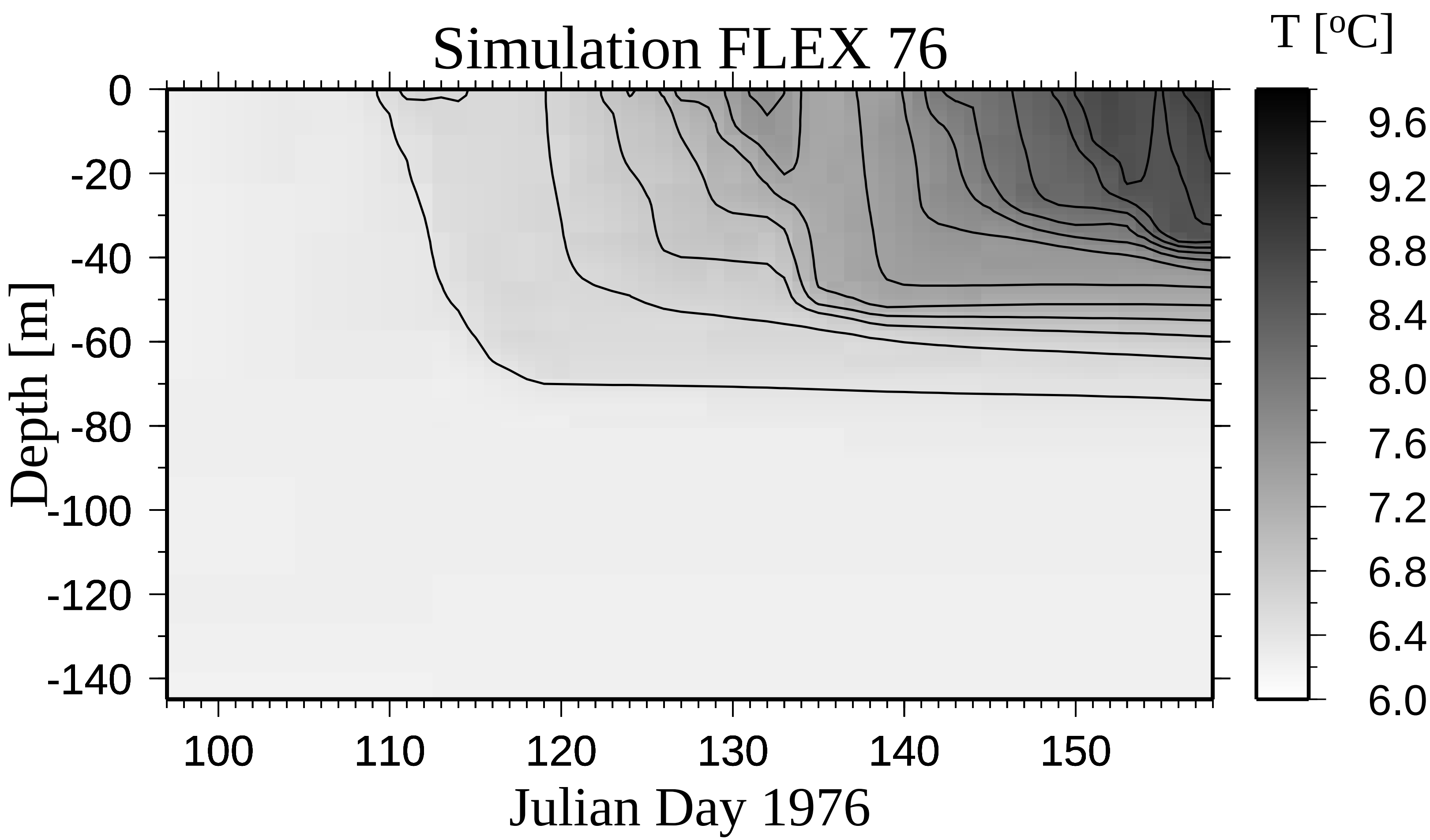
$$k^{grid} = \frac{cD}{T^{grid}} \left(c_\rho K_\rho^{grid} + c_u K_u^{grid} + c_d K_d^{grid} + c_b K_b^{grid} \right).$$

Influence of stratification, shear, surface, background:

$$K_\rho^{grid} = \partial_z \rho, \quad K_u^{grid} \propto \partial_z u, \quad K_d^{grid} \propto (\eta + z)^{-1}, \quad K_b^{grid} \propto D^{-1}.$$

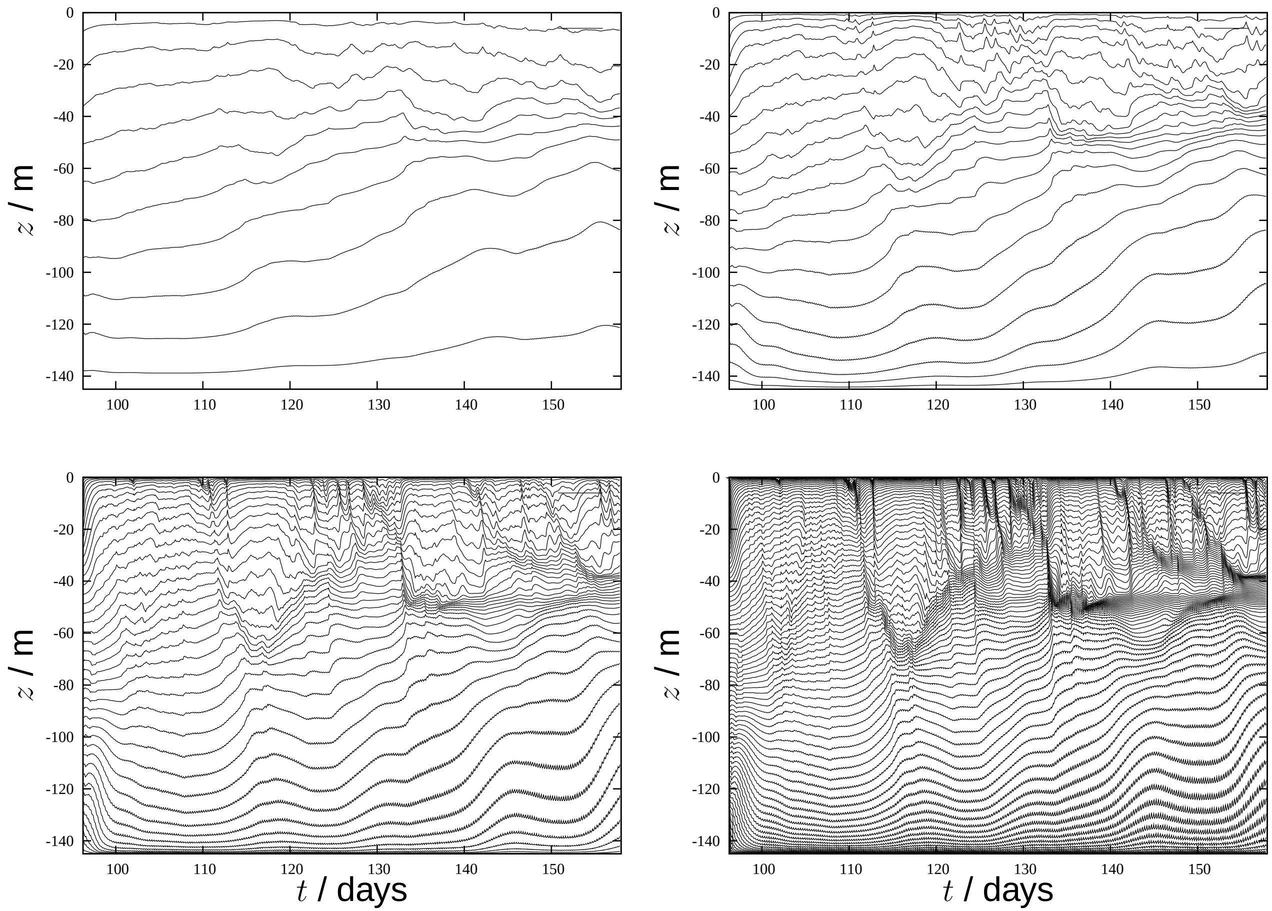
For details, see Burchard and Beckers [2004].

1D simulation North Sea



1D simulation North Sea

Layer distribution for 10, 20, 40 and 80 layers:



Conclusion by Burchard and Beckers [2004]:

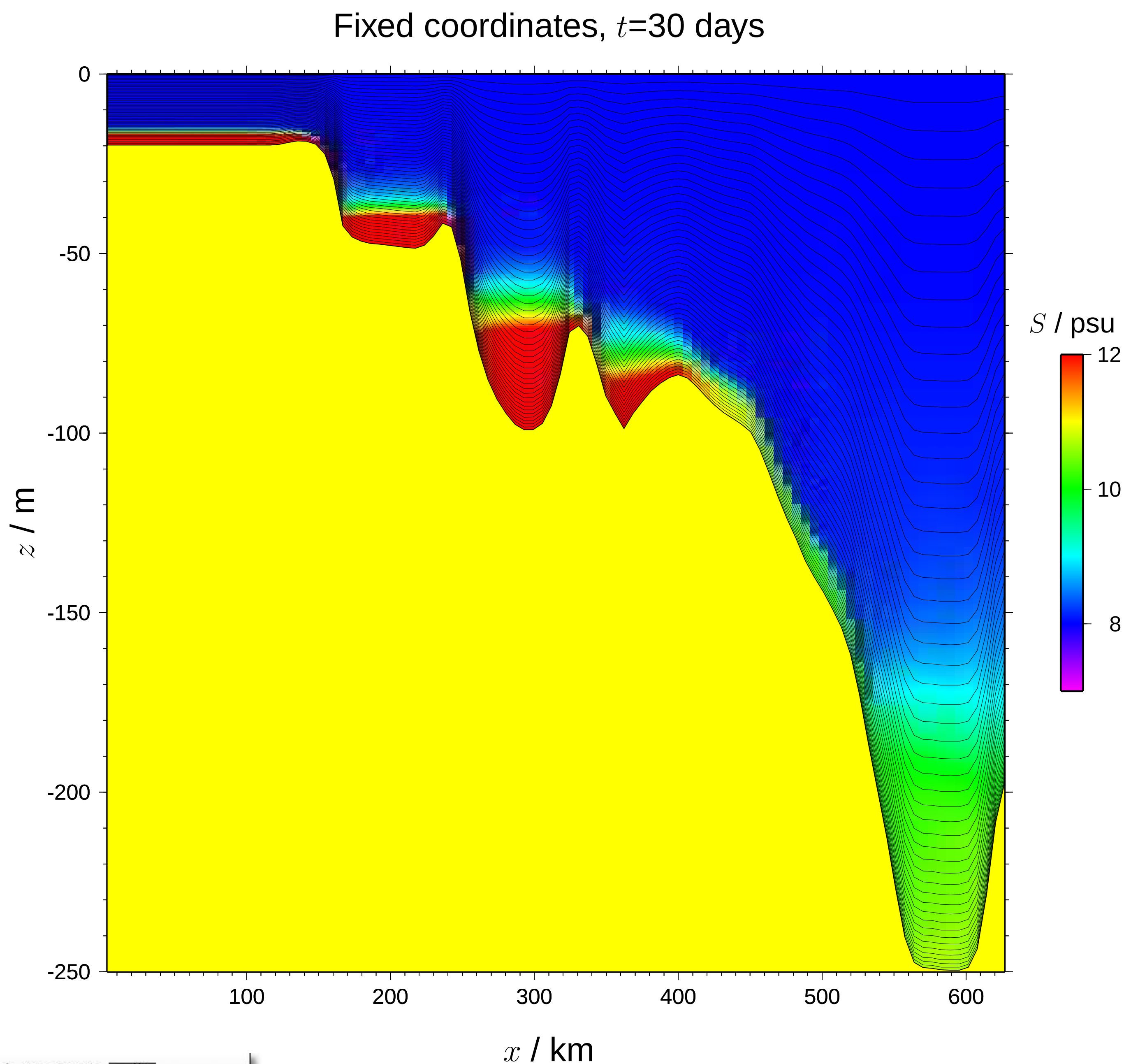
For fixed number of layers adaptive grids may be more accurate.

2D and 3D Adaptive grids

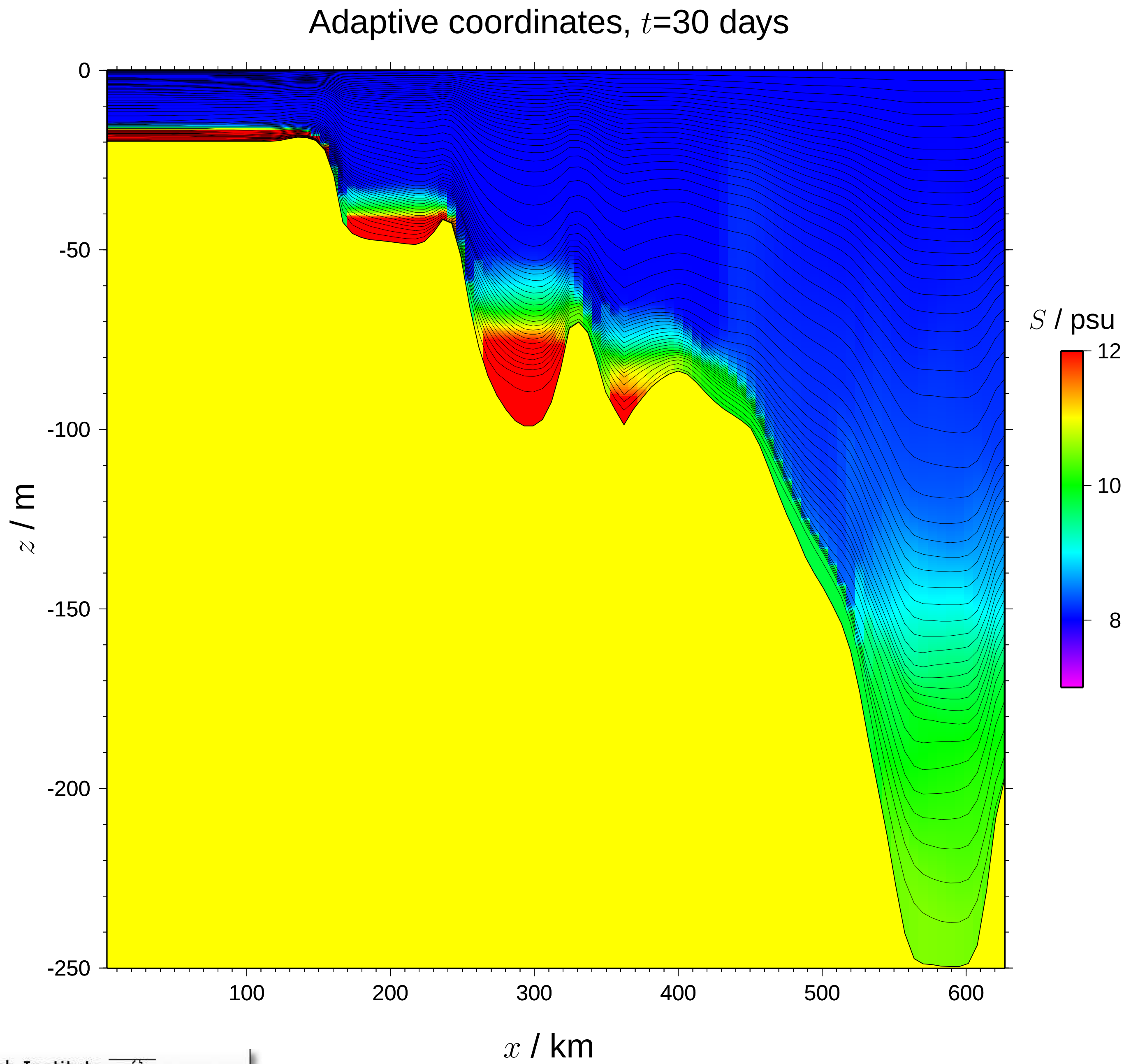
Same strategy as for 1D will be used with some additional measures:

- Horizontal diffusion of layer thickness
- Horizontal diffusion of layer interface position
- Lagrangian tendency
- Isopycnal tendency

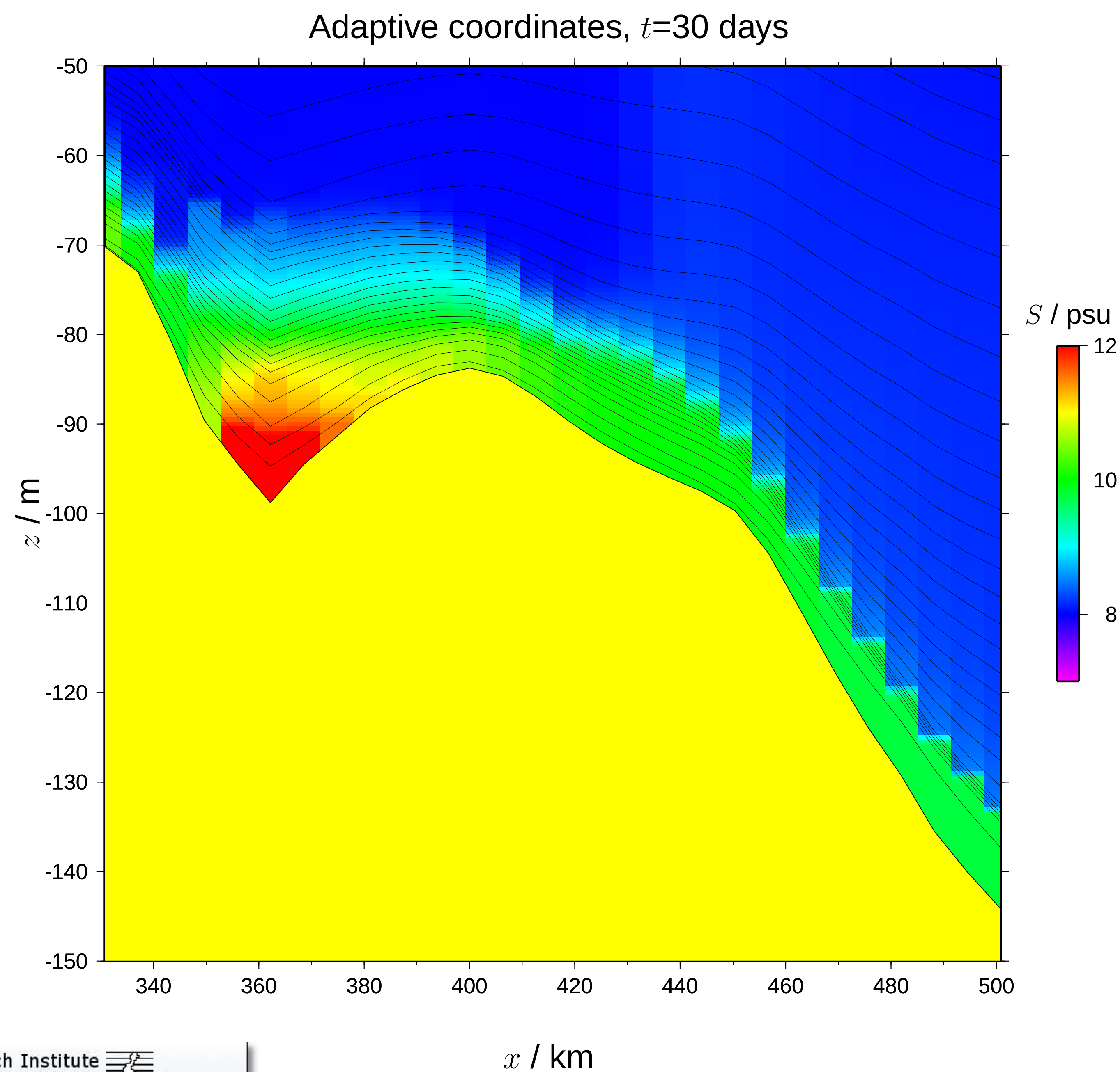
2D inflow experiment



2D inflow experiment



2D inflow experiment



Conclusions

- The approach presented here is similar to isopycnal coordinates, but more general.
- Implementation of the proposed adaptive grid strategy into models with general vertical coordinates (such as ROMS, GETM) is straight-forward.
- Ocean (and atmosphere) models using vertically adaptive grids may give more accurate numerical approximations than models with fixed grids.
- Much more development work is necessary for optimising and generalising this adaptive grid approach.