Lagrangian Particle Modelling in Inhomogeneous Turbulent Flow

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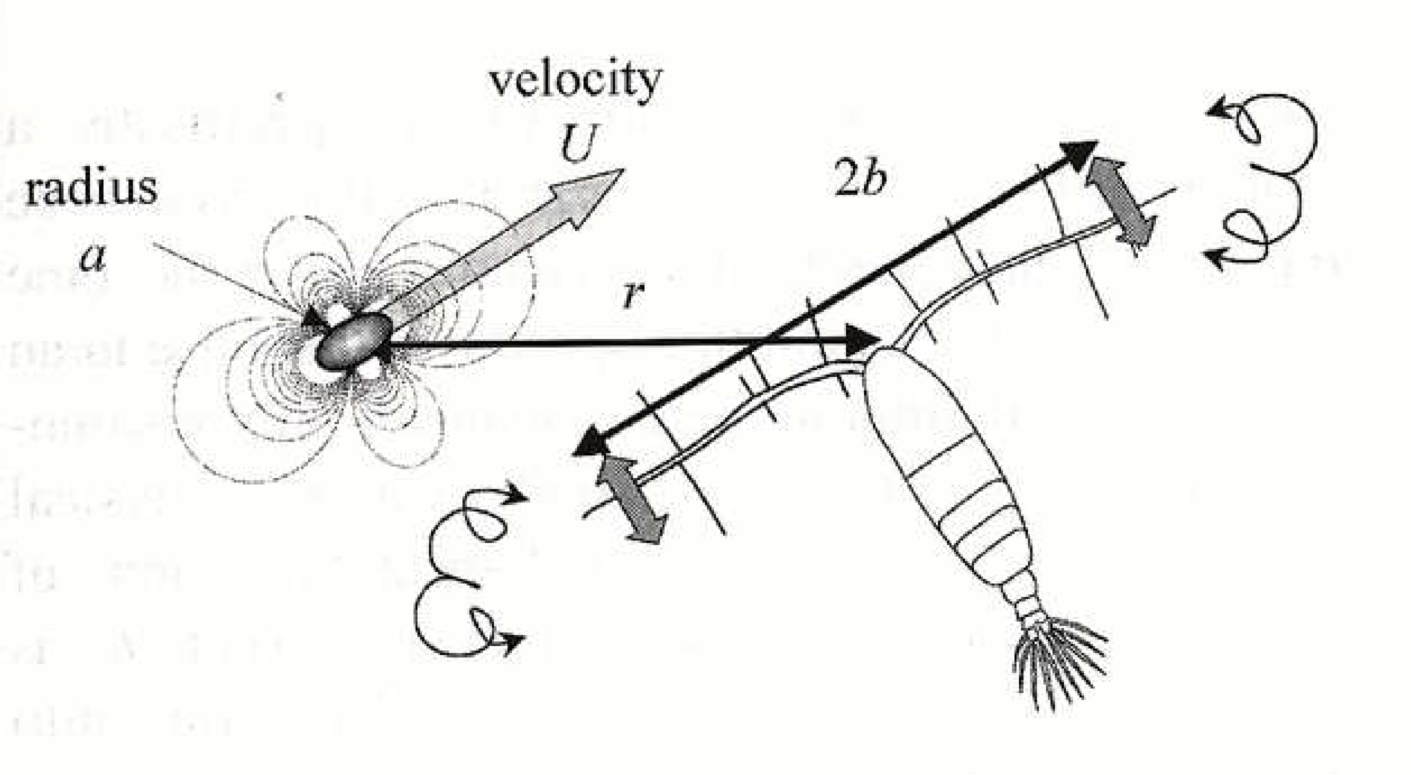
Contents

- Random walk consistent with diffusion
- Basic test: homogeneous concentration
- Less basic test: Rouse profile
- Advanced test: mussel filtration
- Project sketch: AlgaLag
- First results for AlgaLag: Photo response model
- No conclusions yet

Motivation

Complex ***particle-particle*** **interaction:**

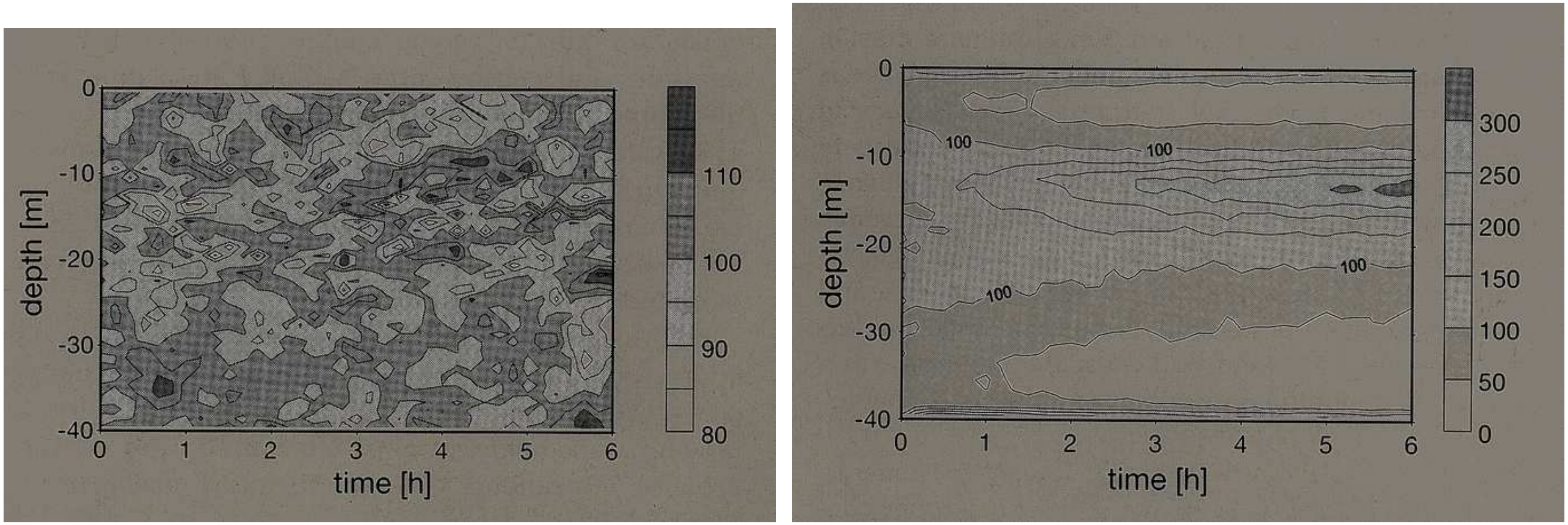


Any chance to model this for a water column ?

Reference: Visser and Stips [2002]

Motivation

Consistent (left) and inconsistent (right) random walk:



(Based on viscosity profile with surface and bottom mixed layer and minimum at $z = -10$ m)

Reference: Visser [1997]

Random walk

Random walk consistent with diffusion equation:

$$\partial_t C - \partial_z \left(\nu_t \partial_z C\right) = 0. \tag{1}$$

$$\begin{aligned} z_i^{n+1} &= z_i^n + \partial_z \nu_t(z_i^n)\Delta t \\ &+ R\left\{2r^{-1}\nu_t(z_i^n + \tfrac{1}{2}\partial_z \nu_t(z_i^n)\Delta t)\Delta t\right\}^{1/2} \end{aligned} \tag{2}$$

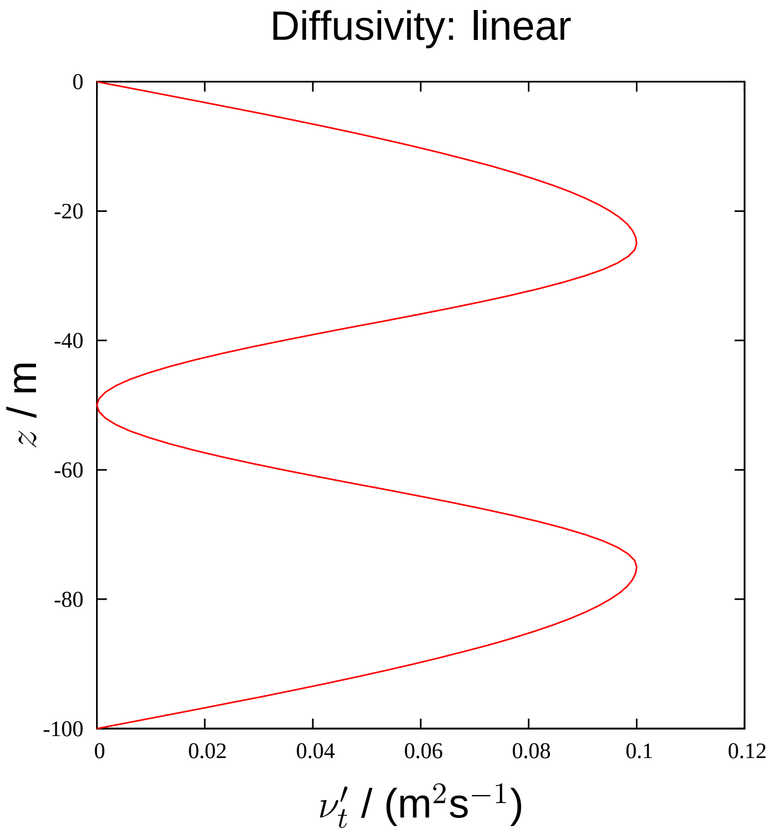
R: random process with $\langle R\rangle = 0$ and $\langle R^2\rangle = r$.

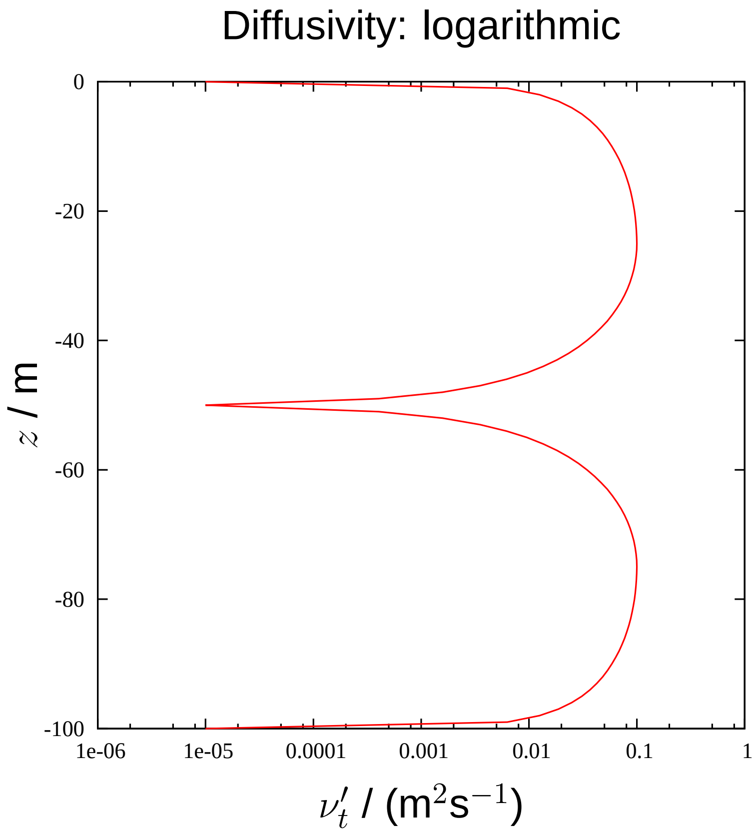
z_i^n : vertical position of particle i at time step n.

Reference: Visser [1997]

Homogeneous concentration

Prescribed diffusivity profile

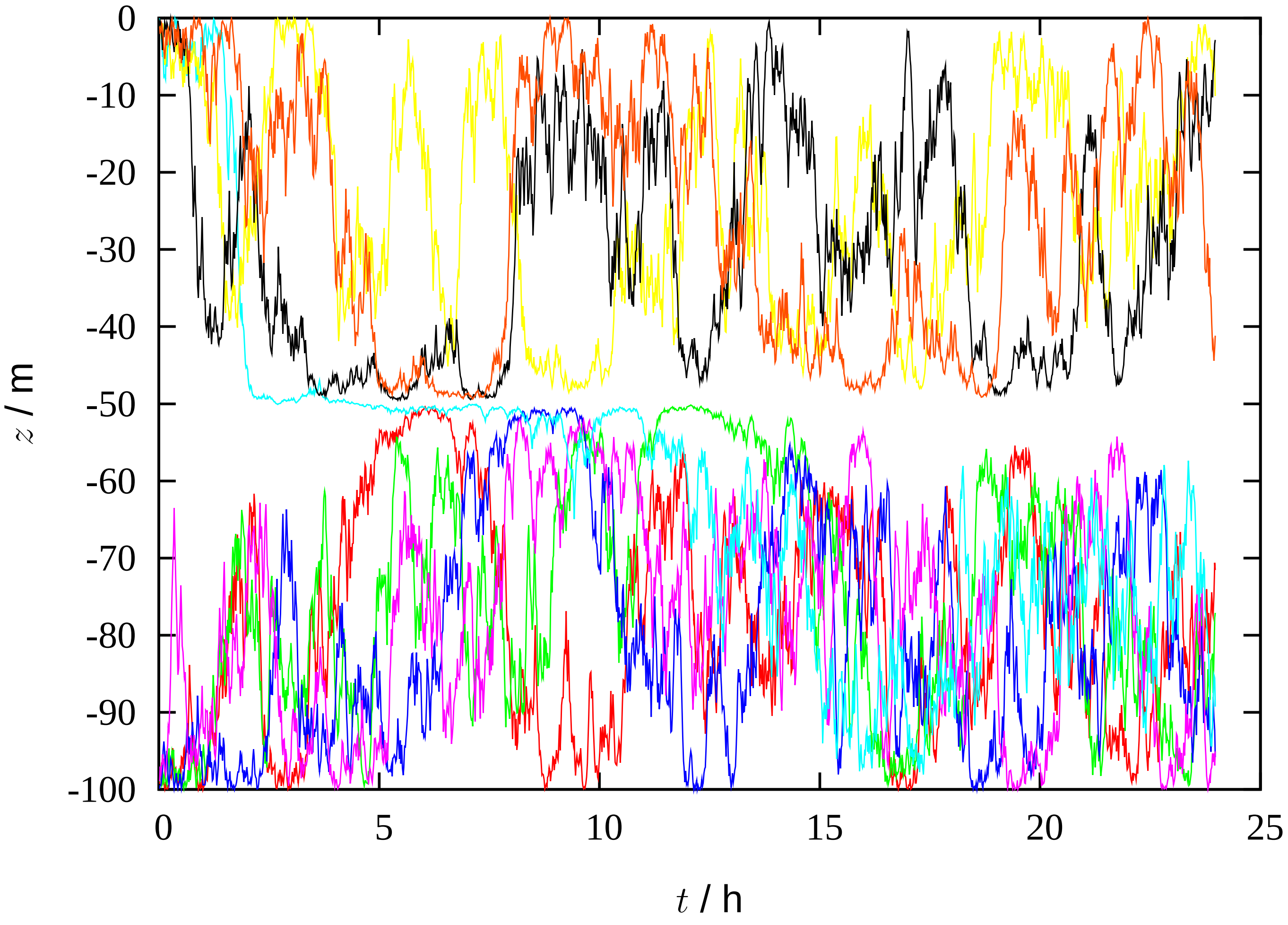




Homogeneous concentration

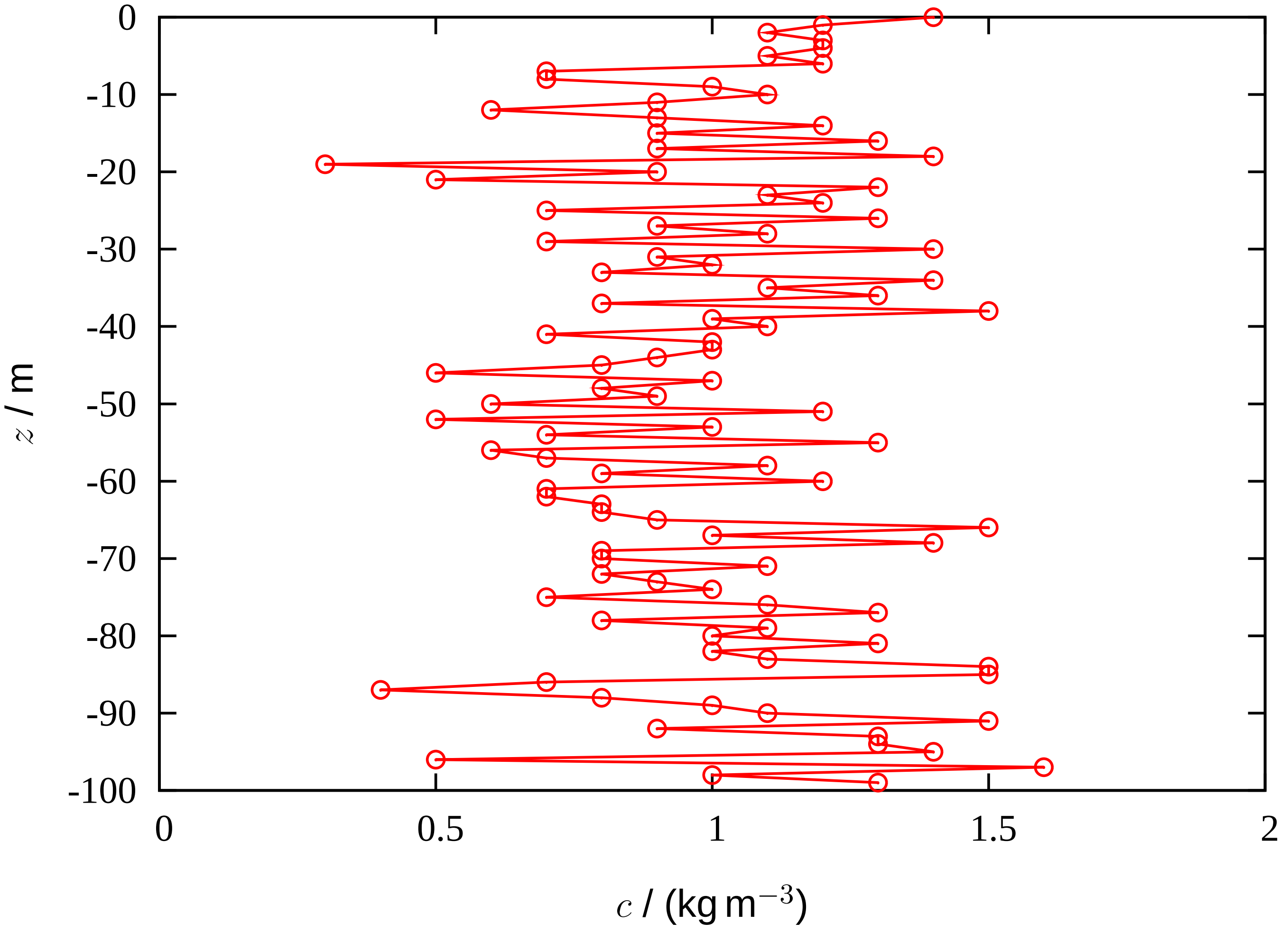
Some trajectories

Starts at: z=-0.1; -0.2; -0.3; -0.4; -99.6; -99.7; -99.8; -99.9 m



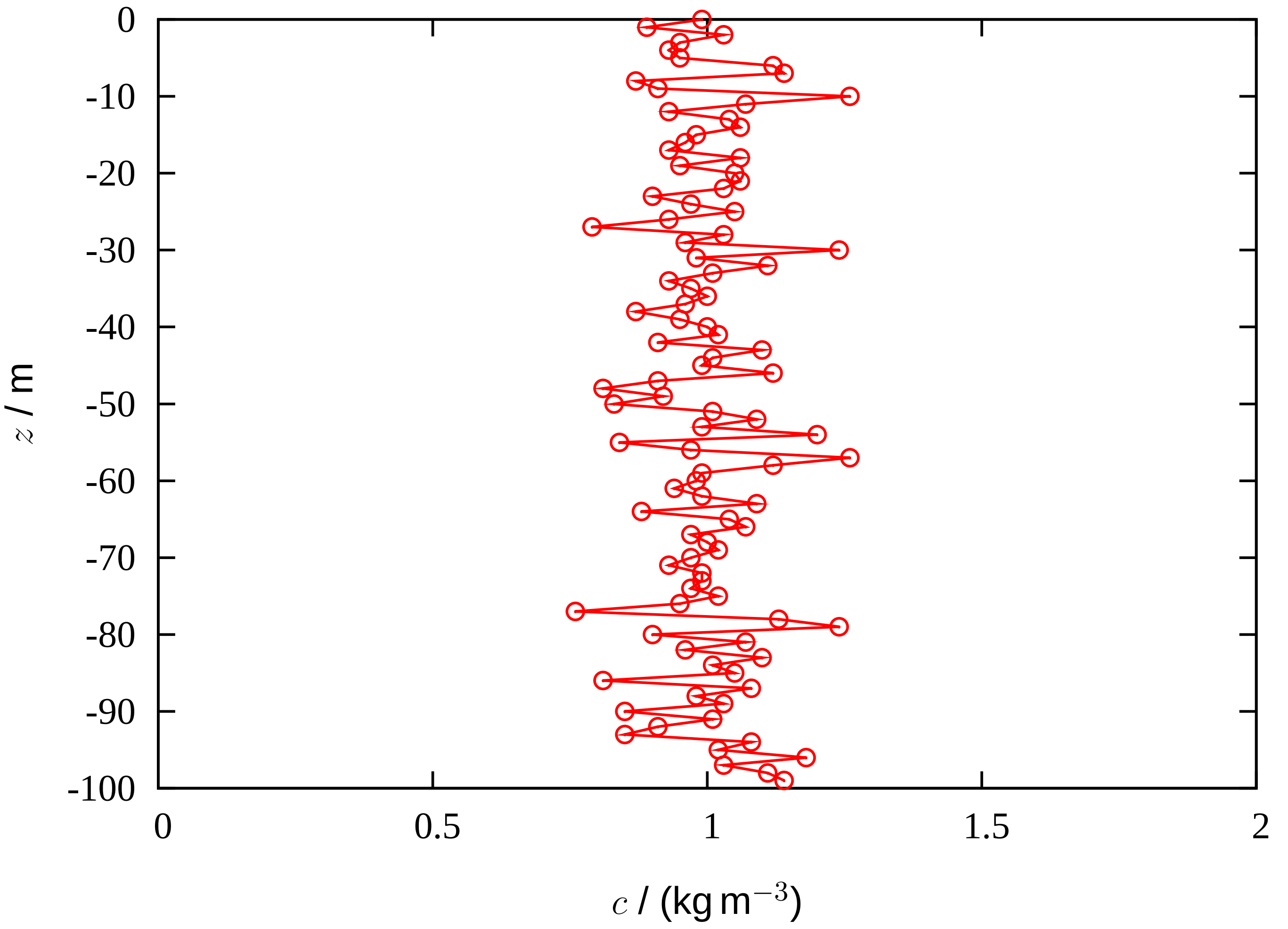
Homogeneous concentration

Concentrations for $N = 1000$ particles at t=24 h



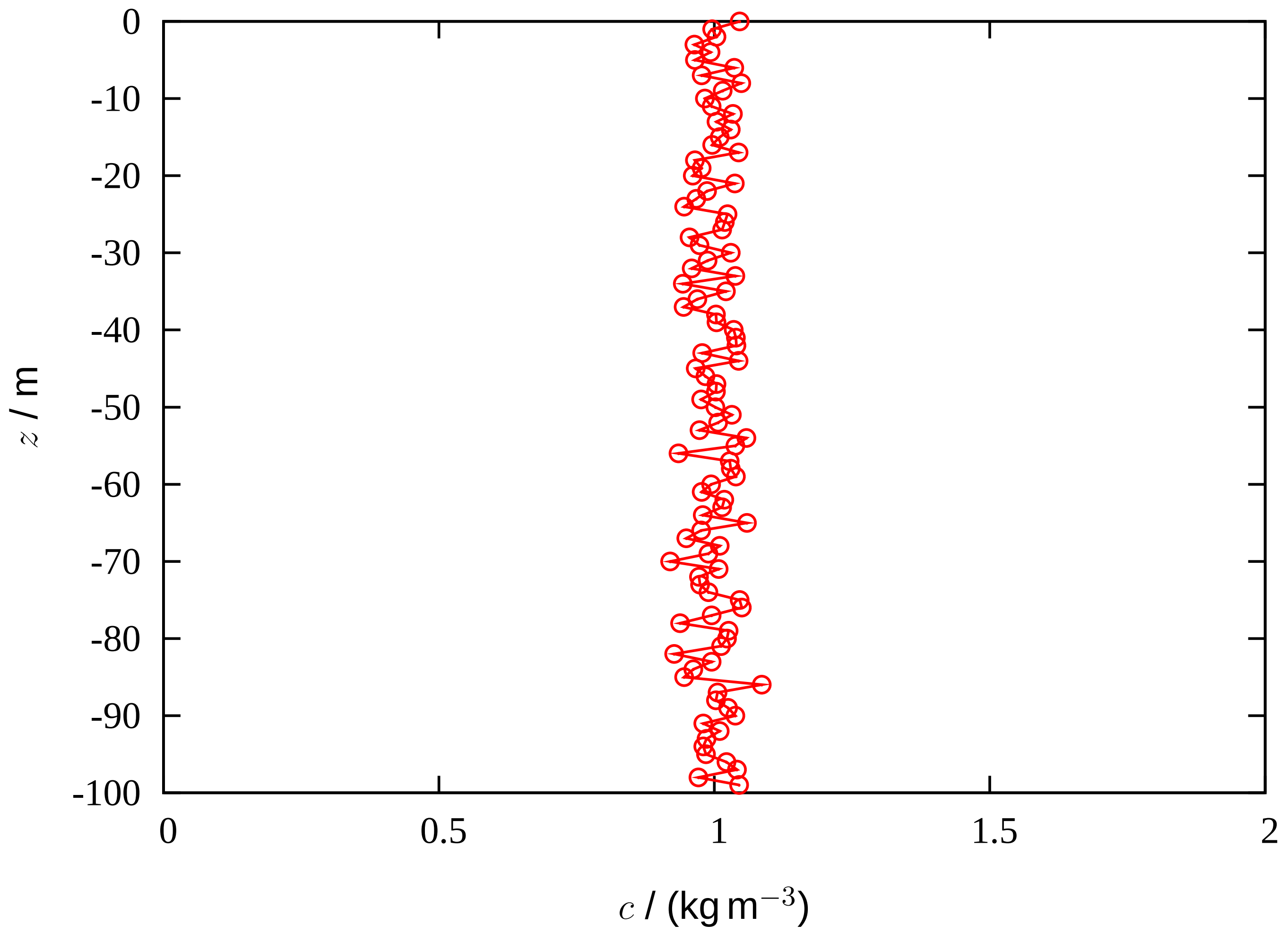
Homogeneous concentration

Concentrations for $N = 10000$ particles at t=24 h



Homogeneous concentration

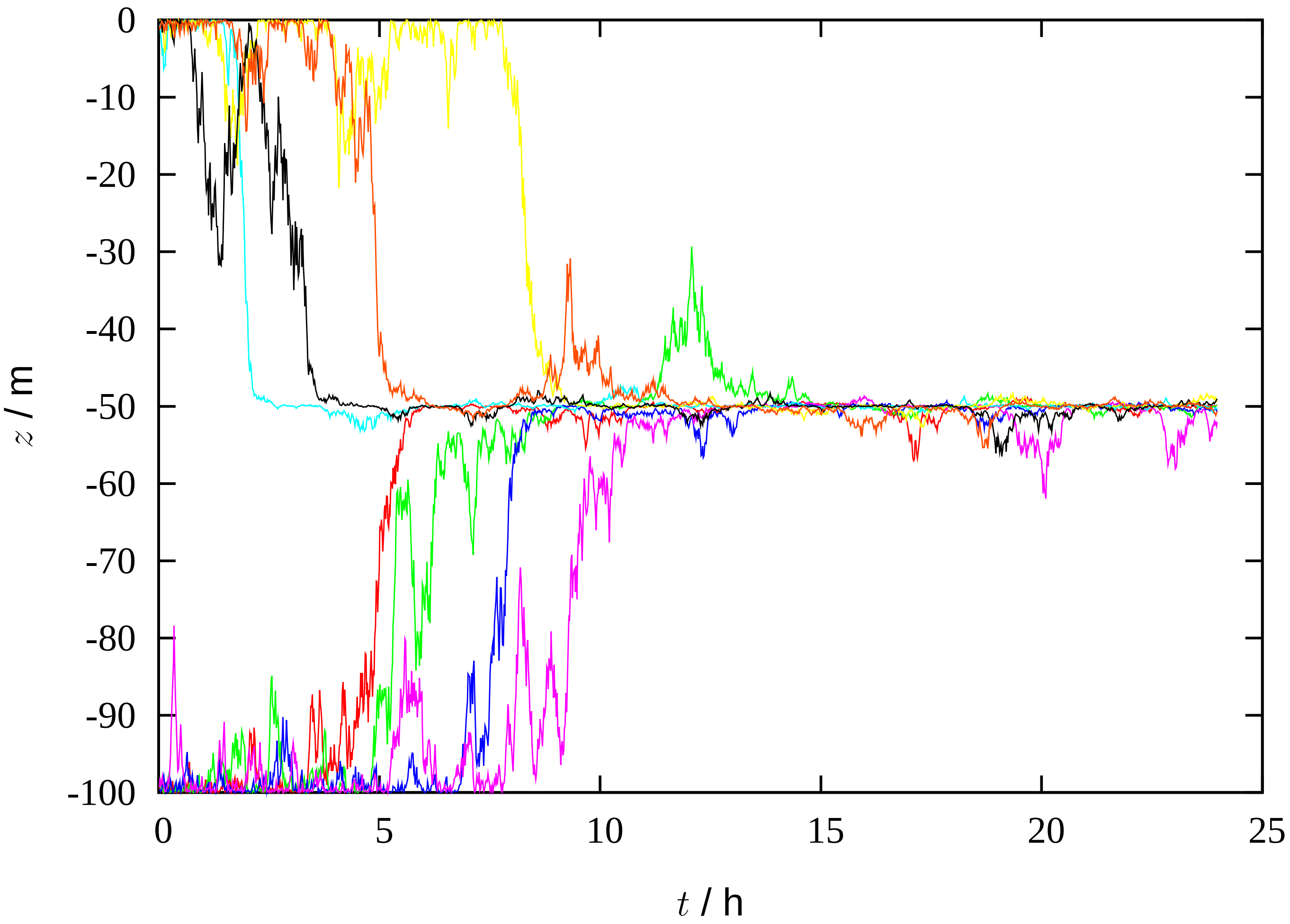
Concentrations for $N = 100000$ particles at t=24 h



Homogeneous concentration

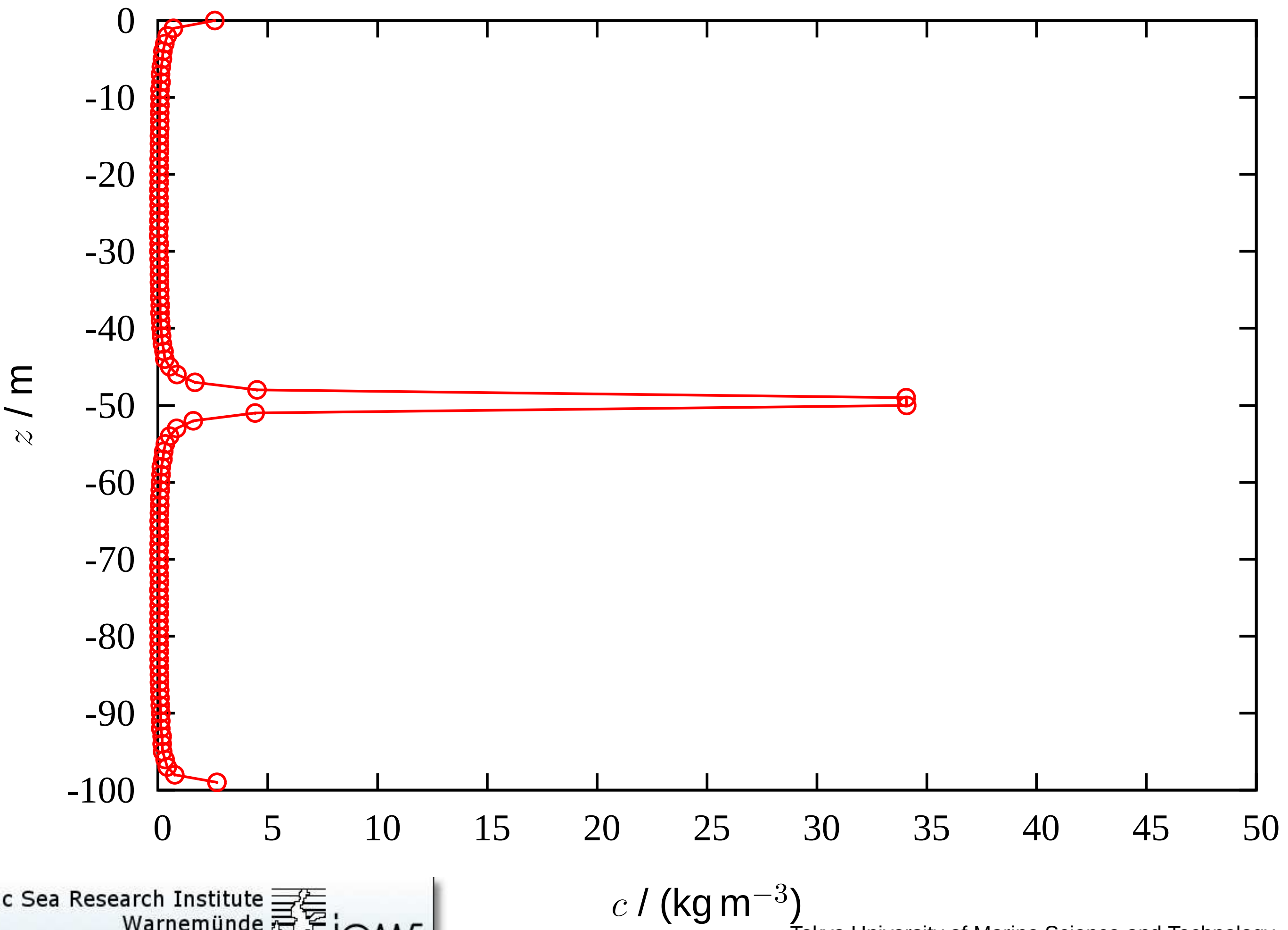
Some trajectories (inconsistent random walk)

Starts at: z=-0.1; -0.2; -0.3; -0.4; -99.6; -99.7; -99.8; -99.9 m



Homogeneous concentration

Concentrations for $N = 100000$ particles at t=24 h (inconsistent random walk)



Homogeneous concentration

Reason for failure of naive random walk

Naive random walk is consistent with (Visser [1997])

$$\partial_t C - \partial_{zz} \left(\nu_t C\right) = 0 \tag{3}$$

which is equivalent to

$$\partial_t C - \partial_z(\partial_z \nu_t C) - \partial_z \left(\nu_t \partial_z C\right) = 0. \tag{4}$$

Thus, an advection with the advective velocity $-\partial_z \nu_t$ (against the viscosity gradient) is performed.

Rouse profile

Analytical Problem

- Constant settling velocity w_c
- Parabolic eddy diffusivity ν_t
- Reflective bottom and surface
- Steady-state solution

$$\partial_t C + \partial_z \left(w_c C - \nu_t \partial_z C \right) = 0, \tag{5}$$

with

$$\nu_t = \kappa u_* (-z) \frac{D + z_0 + z}{D + z_0}. \tag{6}$$

Rouse profile

Analytical Solution

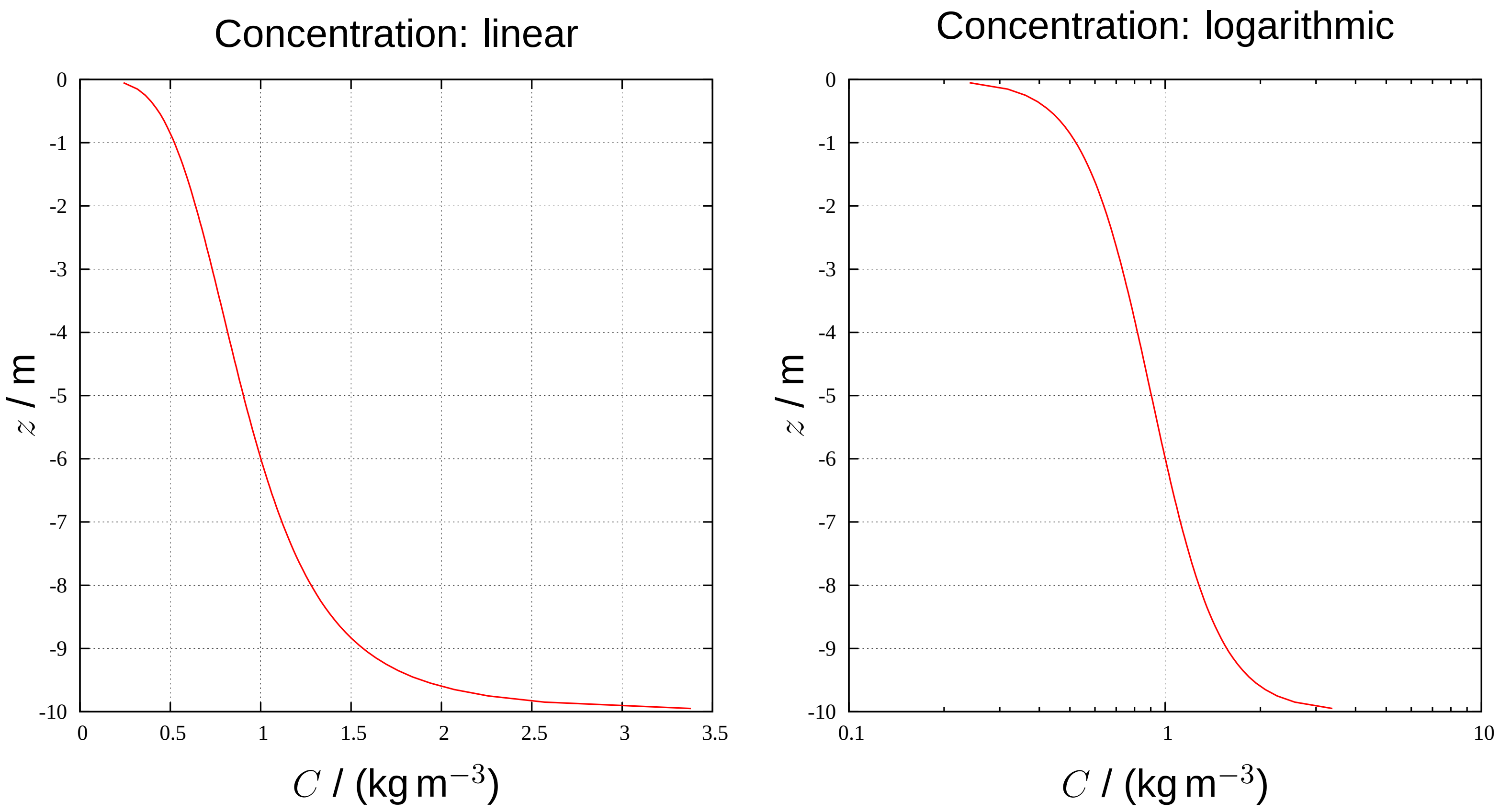
$$\frac{C}{C_0} = \left(\frac{-z}{D + z_0 + z} \right)^{-w_c/(\kappa u_*)} \tag{7}$$

Rouse number:

$$R = \frac{-w_c}{u_*}. \tag{8}$$

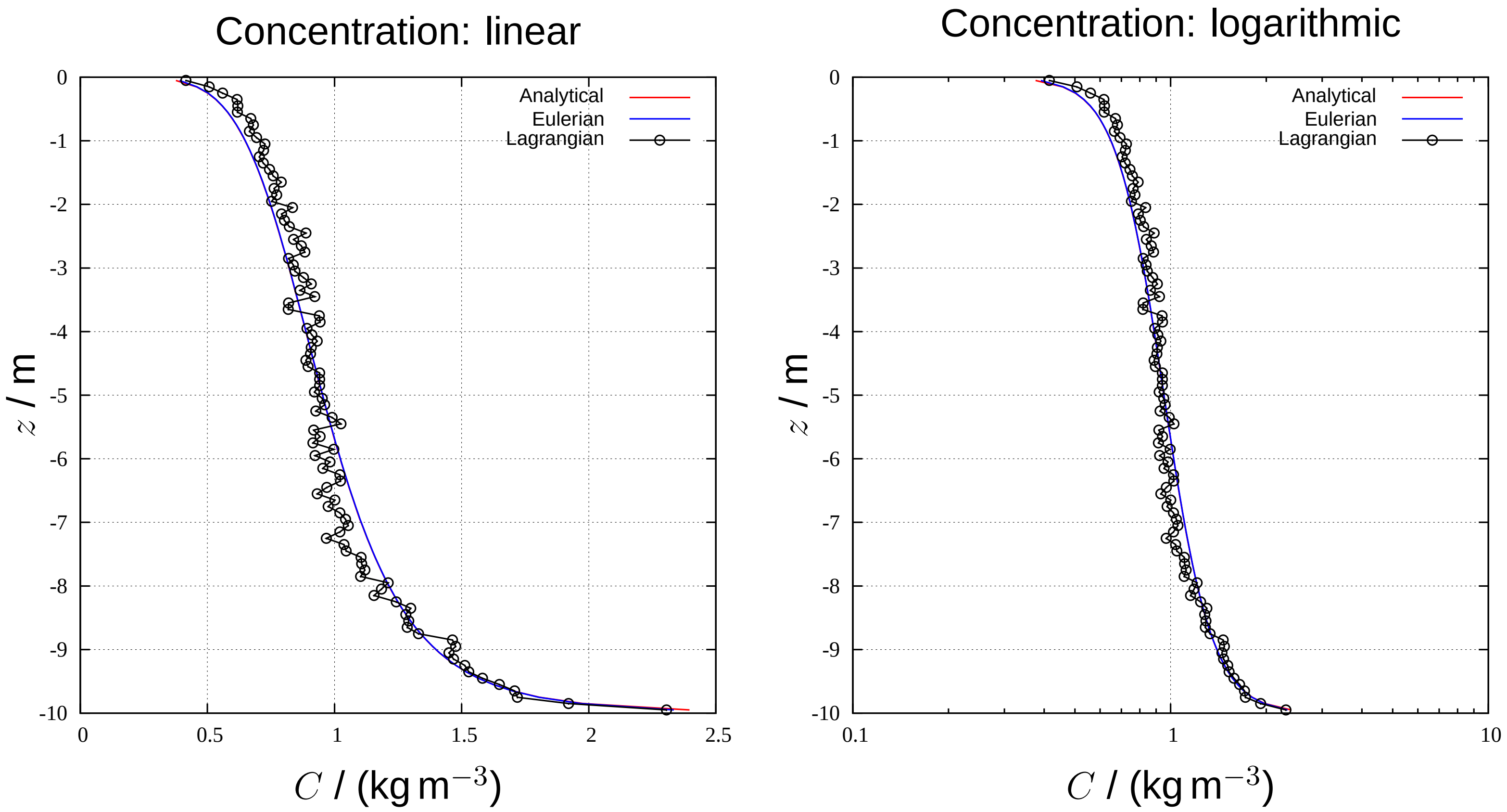
Rouse profile

Analytical Solution



Rouse profile

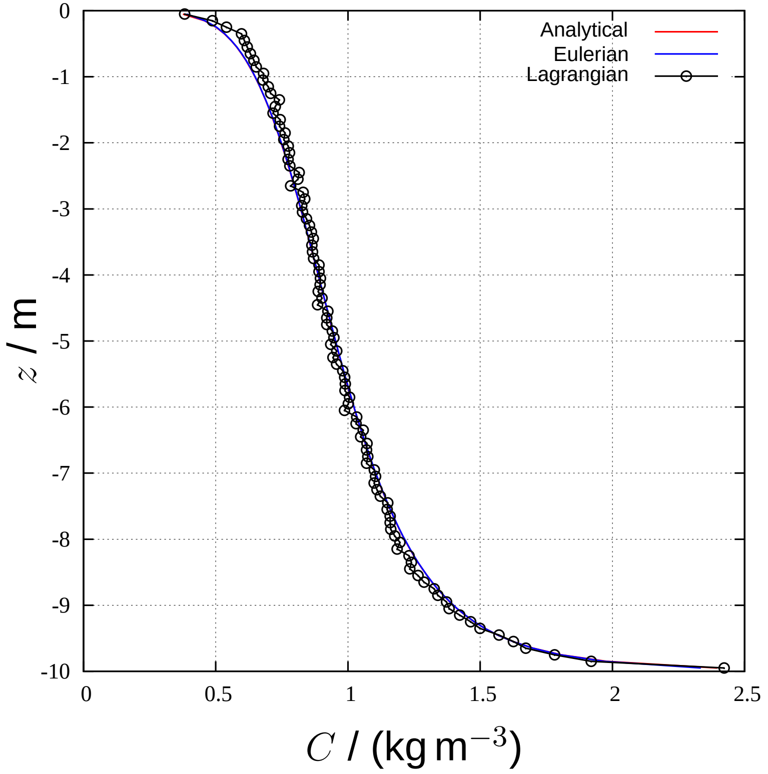
Numerical Solution for N = 1000 Particles



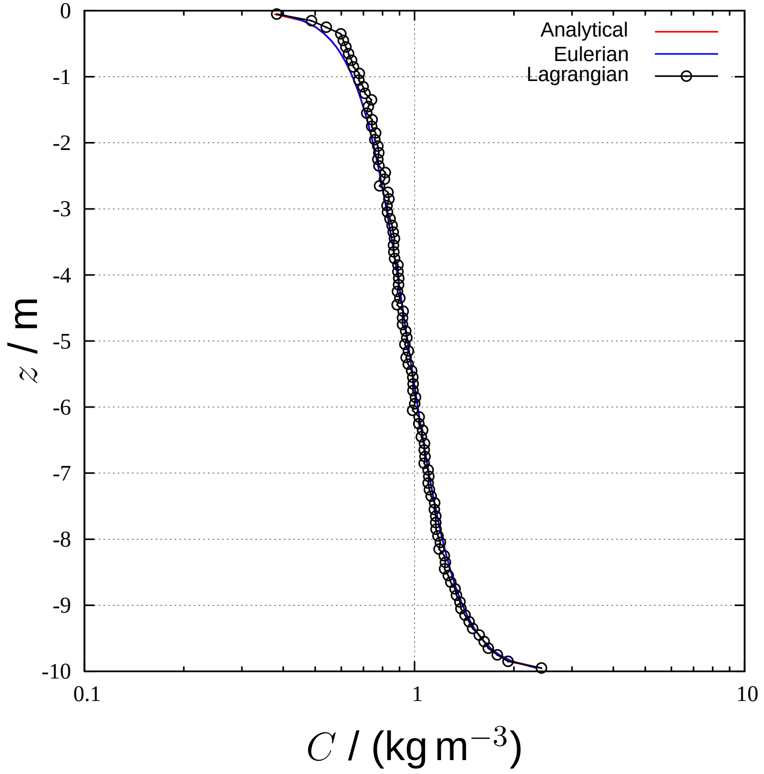
Rouse profile

Numerical Solution for N = 10000 Particles

Concentration: linear

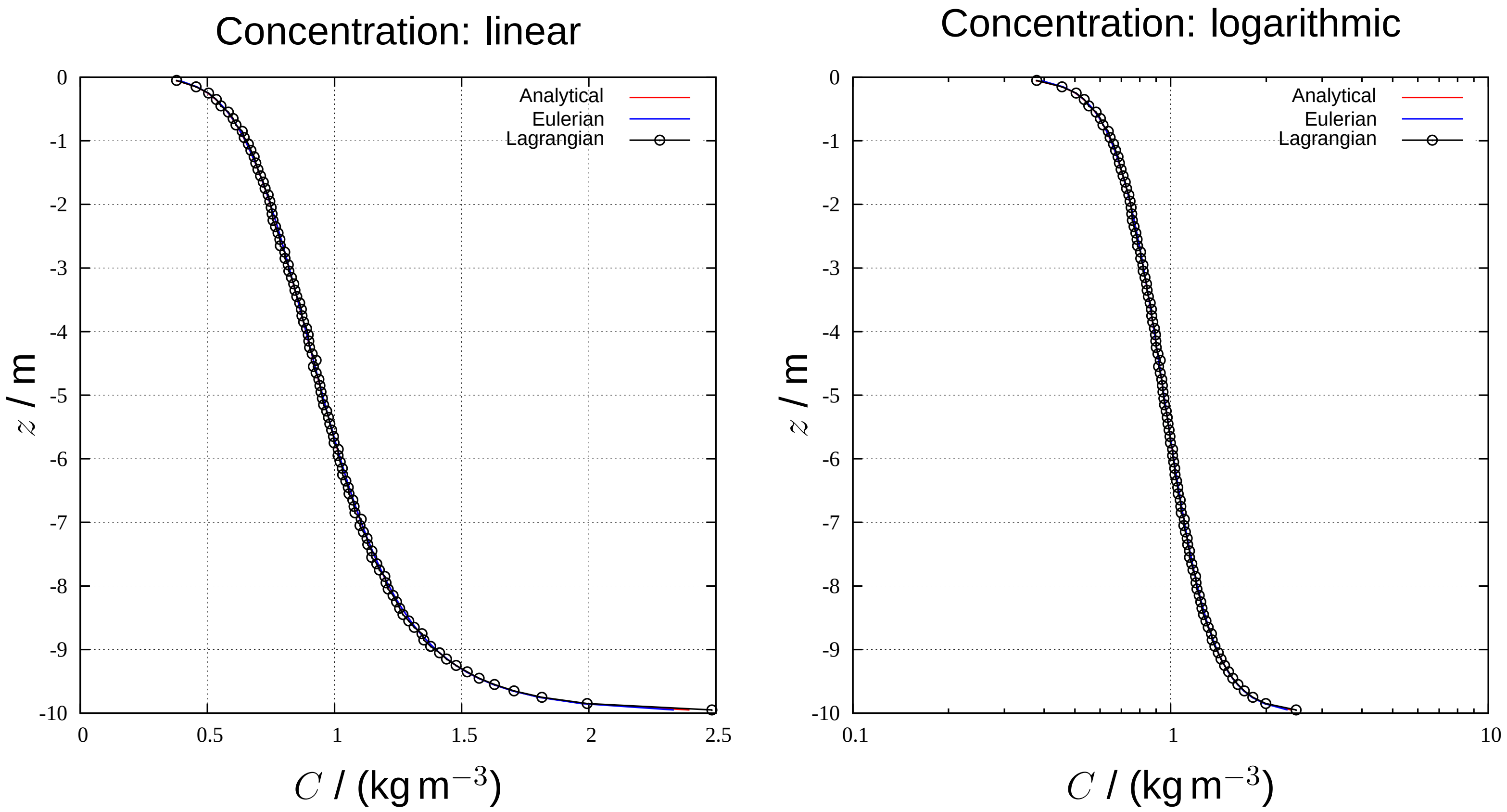


Concentration: logarithmic



Rouse profile

Numerical Solution for N = 100000 Particles



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Warnemünde

Rouse profile

Error with respect to analytical solution (standard deviation)

Lagrangian scheme

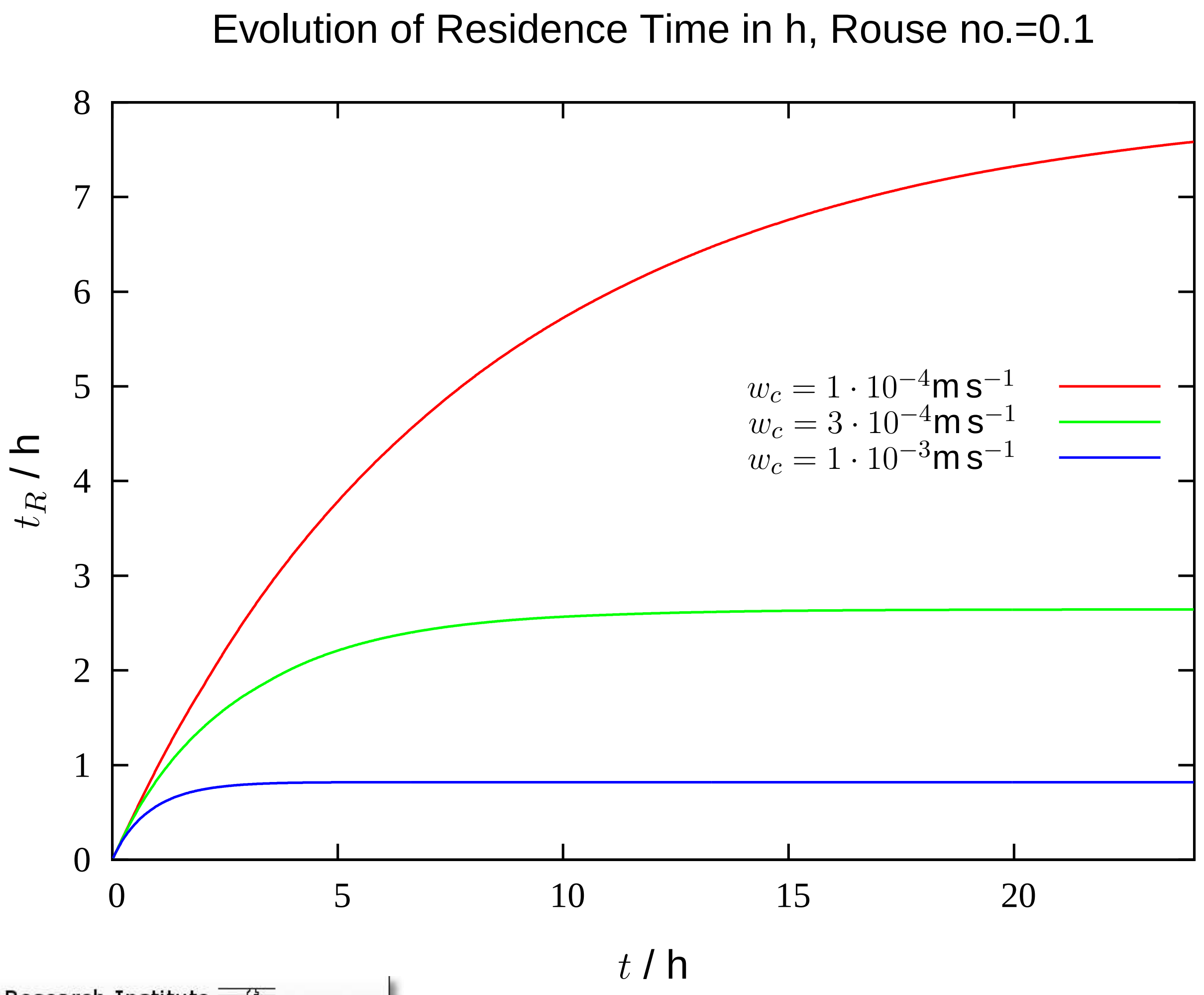
N	10^3	10^4	10^5
σ	0.059	0.023	0.010

Eulerian scheme

Advection scheme	upwind	TVD
σ	0.011	0.0064

Residence time

Residence time in water column (above $z = -9$ m)
Start at $z = 5$ m, $N = 10000$



Residence time

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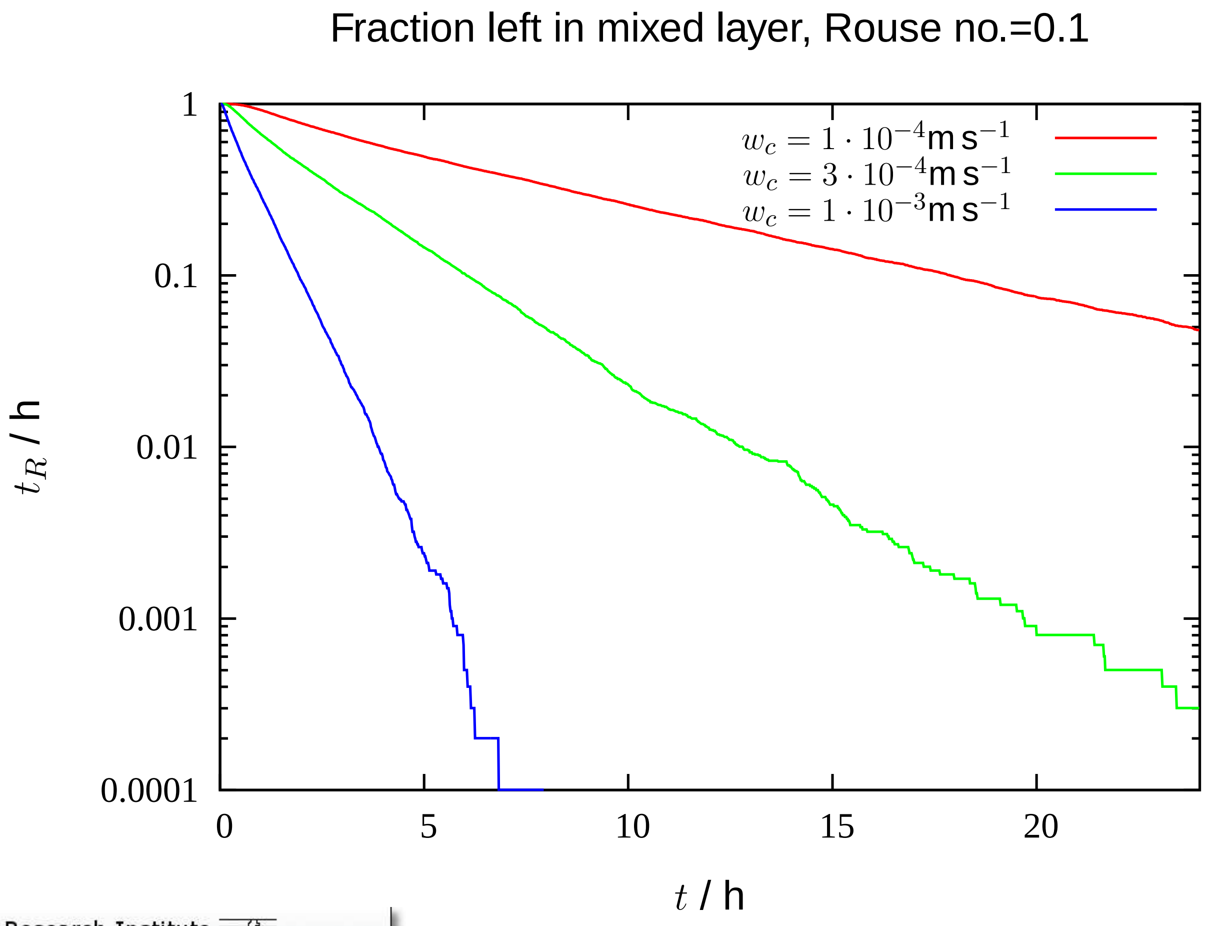


Mussel filtration

Benthic filter feeders as sinks for suspended particulate matter

Eulerian formulation of boundary condition:

$$\nu_t \partial_z C\Big|_{z=-H} = w_f C\Big|_{z=-H}, \quad w_f = N_m V_f \tag{9}$$

w_f: filtration velocity

N_m: number of mussels per m^2 (e.g. 1000)

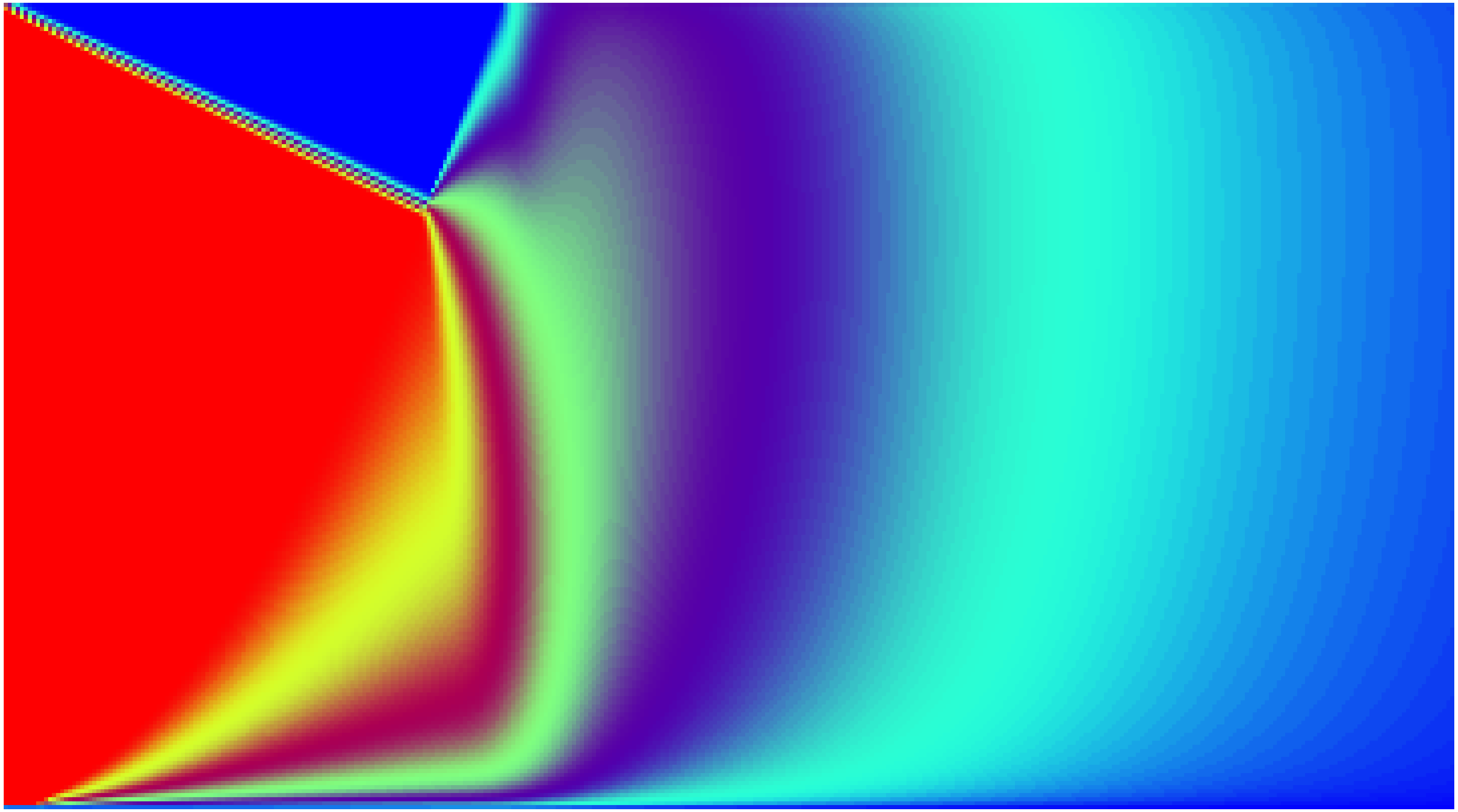
V_f: filtration volume per mussel (e.g. 18 l per hour)

Lagrangian formulation of boundary condition:

At each time step take out all particles which are below $z = -H + \Delta t w_f$.

Mussel filtration

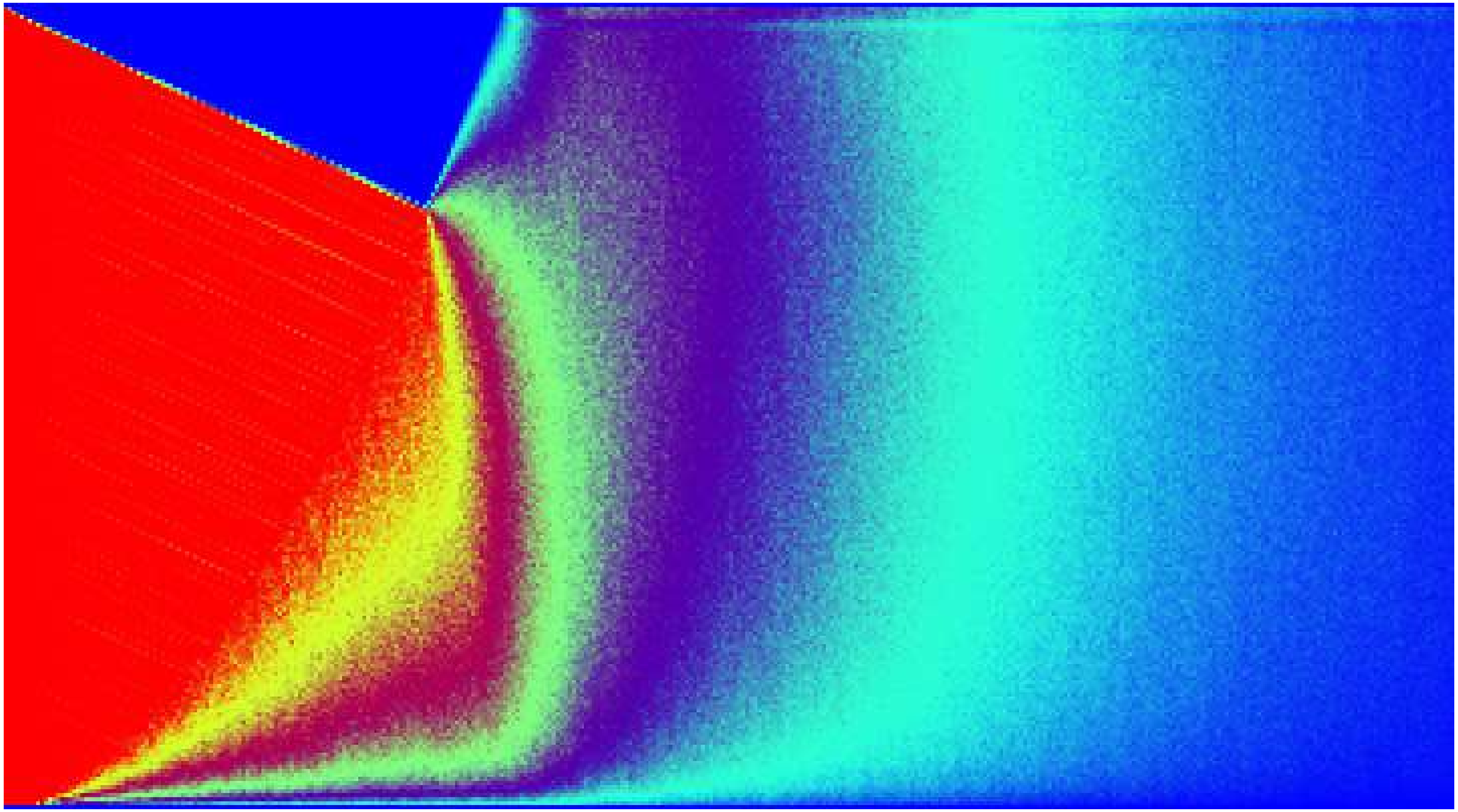
Profiles of suspended matter: Euler



(x-axis: 0h $\leq t \leq$ 12 h, z-axis: -10 m $\leq z \leq 0$ m, contours: 0 $\leq C \leq 1$)

Mussel filtration

Profiles of suspended matter: Lagrange

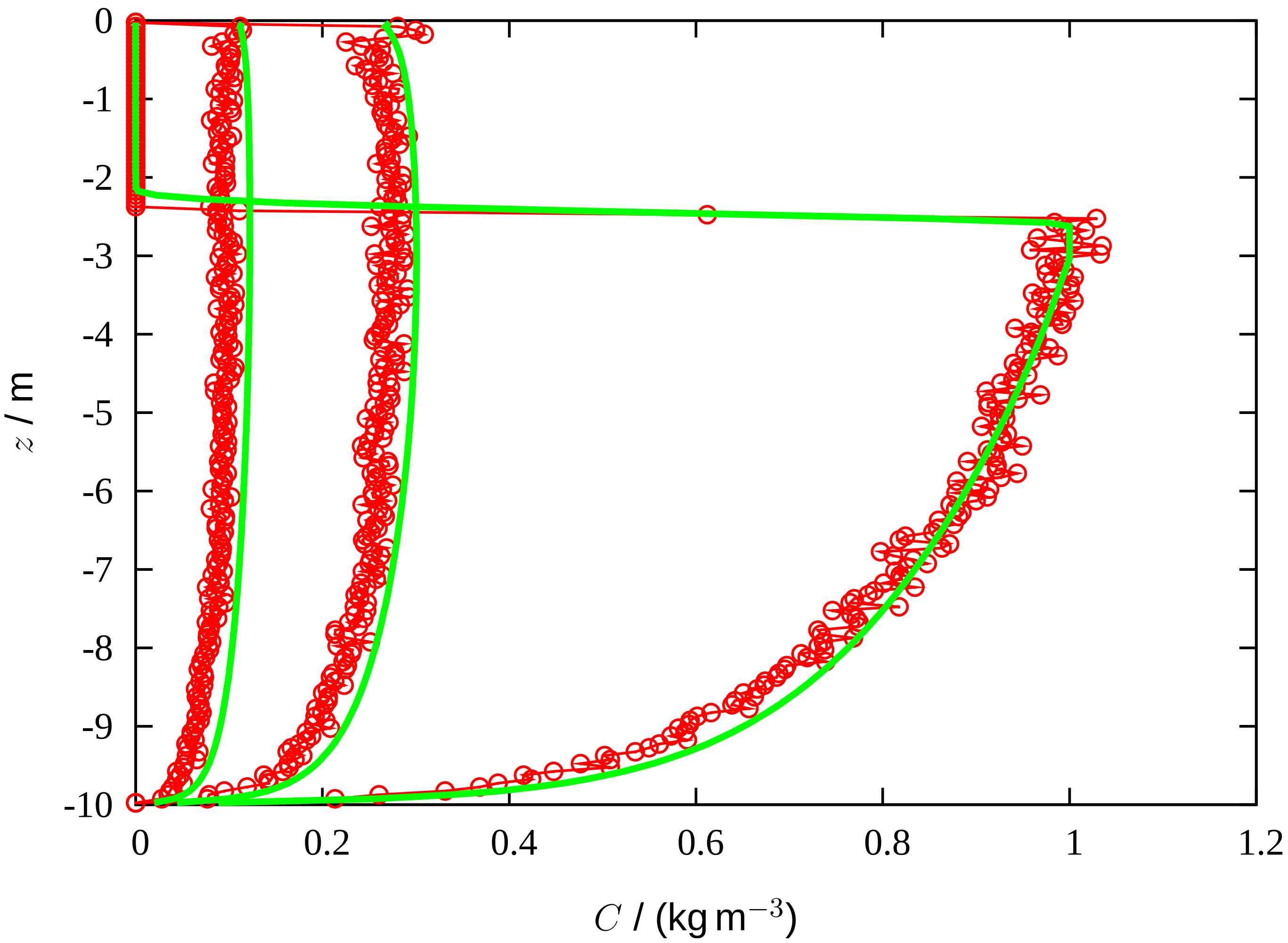


(x-axis: 0h $\leq t \leq$ 12 h, z-axis: -10 m $\leq z \leq 0$ m, contours: 0 $\leq C \leq 1$)

Mussel filtration

Profiles of suspended matter: Euler vs. Lagrange

Profiles at t = 3 1/3; 6 2/3; 10 h



Project: AlgaLag

A Lagrangian Study of Algal Bloom in the Ocean Mixed Layer (AlgaLag)

- German-Japanese Cooperation
- DFG-funding 2004-2007 (Travel expenses to Japan)
- Partner: Prof. Hidekatsu Yamazaki (Tokyo University of Marine Science and Technology)
- Objectives:
  - State-of-the-art Lagrangian model into GOTM
  - Improve random walk models for turbulence interaction
  - Lagrangian model for ecosystem models
  - Include memory effects into Lagrangian model
  - Improved parameterisations for Eulerian models

Photo adaptation

Let any Lagrangian particle represent a phytoplankton cell. Then the photosynthetic production rates may be calculated according to Nagai et al. [2003] as

$$P = P_d + Y(P_l - P_d) \tag{10}$$

with production rates for uninhibited and inhibited cells,

$$\begin{aligned} P_d &= P_{dm}\left(1 - \exp\left(-\frac{PAR}{E_d}\right)\right), \\ P_l &= P_{lm}\left(1 - \exp\left(-\frac{PAR}{E_l}\right)\right), \end{aligned} \tag{11}$$

with P_{dm}, P_{lm}, E_d and E_l being constants.

Photo adaptation, cont’d

Y is the inhibition parameter defined by

$$\frac{\mathrm{d}Y}{\mathrm{d}t} = \frac{1}{\gamma}(X - Y), \tag{12}$$

with the adaptation time scale γ and the instantaneous inhibition parameter

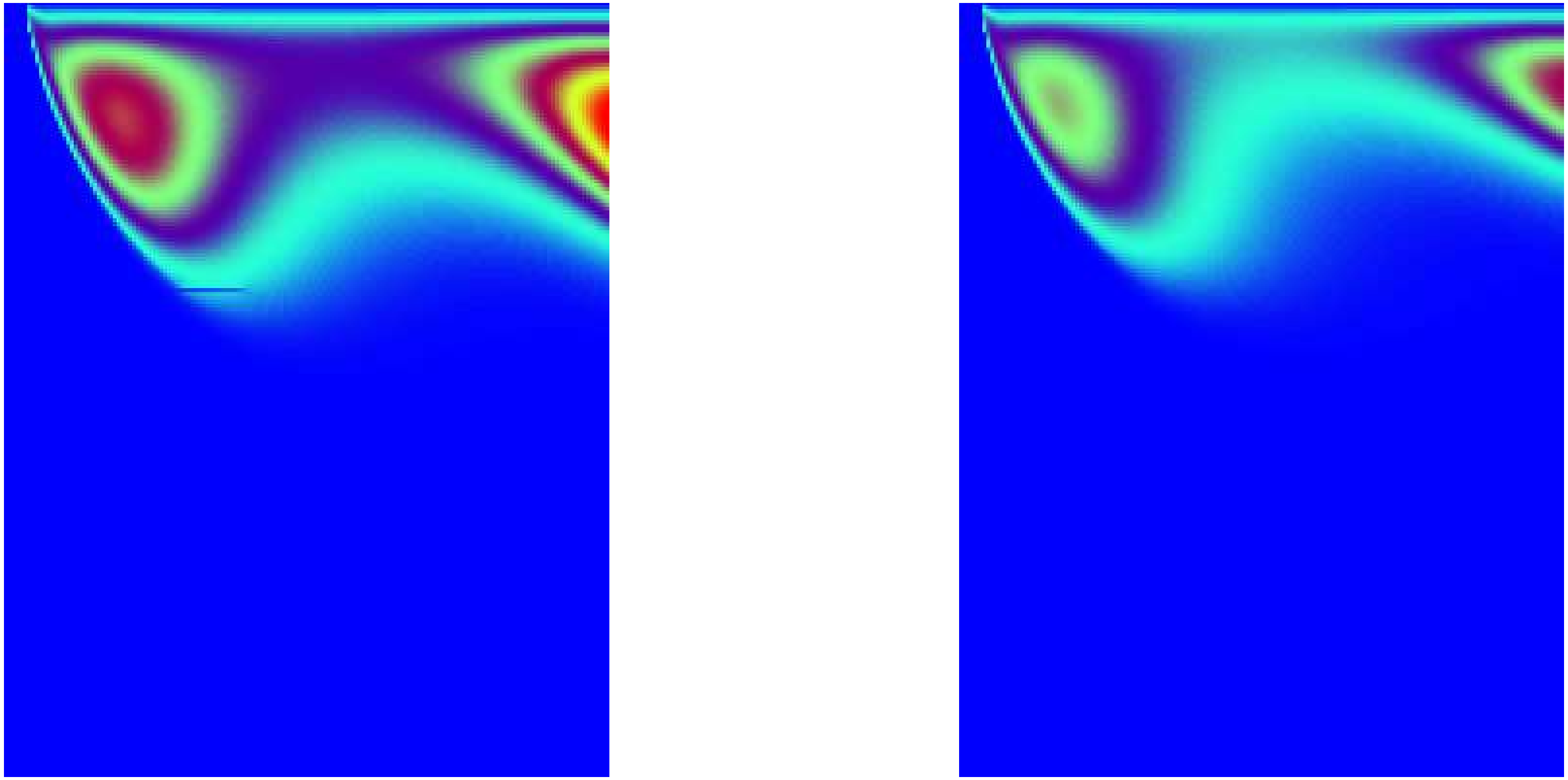
$$X = 1 - \exp\left(-\left(\frac{\max\{PAR, E_b\} - E_b}{E_b}\right)^2\right). \tag{13}$$

PAR is the photosynthetically available radiation.
Some experiments with variations in surface forcing (W = 0, 5, 10 $\mathrm{m\,s^{-1}}$) and turbidity (Jerlov I and III, coastal) have been carried out, see the following slides.

Photo adaptation

W = 10 m s^{-1}, turbidity: Jerlov I (left), coastal (right)

Eddy diffusivity



x-axis: 6:00 h $\leq t \leq$ 19:00 h, z-axis: -100 m $\leq z \leq 0$ m

Contours: 0 $\leq \nu_t' \leq 0.04$ m^{2}s^{-1}

Photo adaptation

W = 5 m s^{-1}, turbidity: Jerlov I (left), coastal (right)

Eddy diffusivity

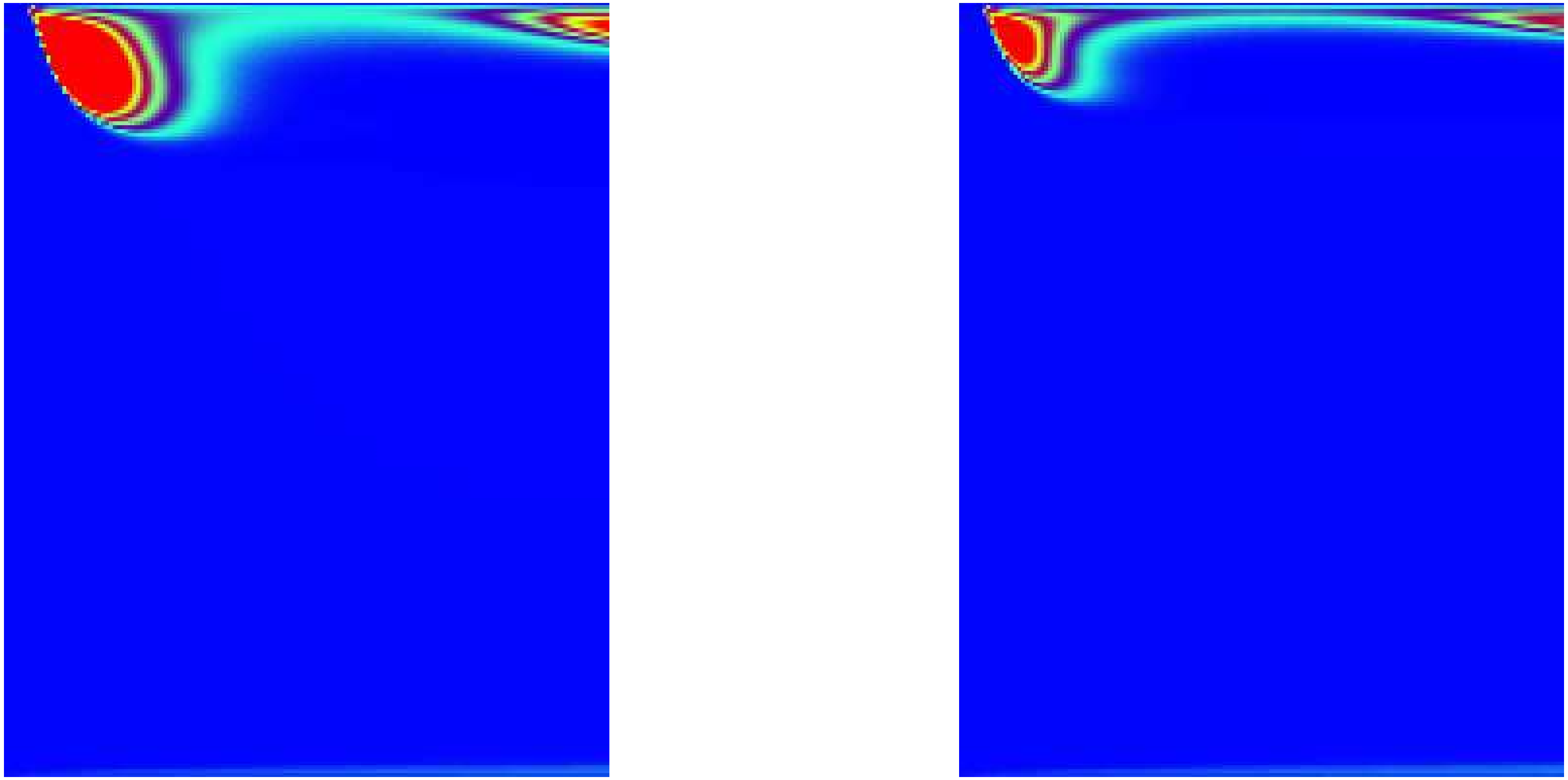


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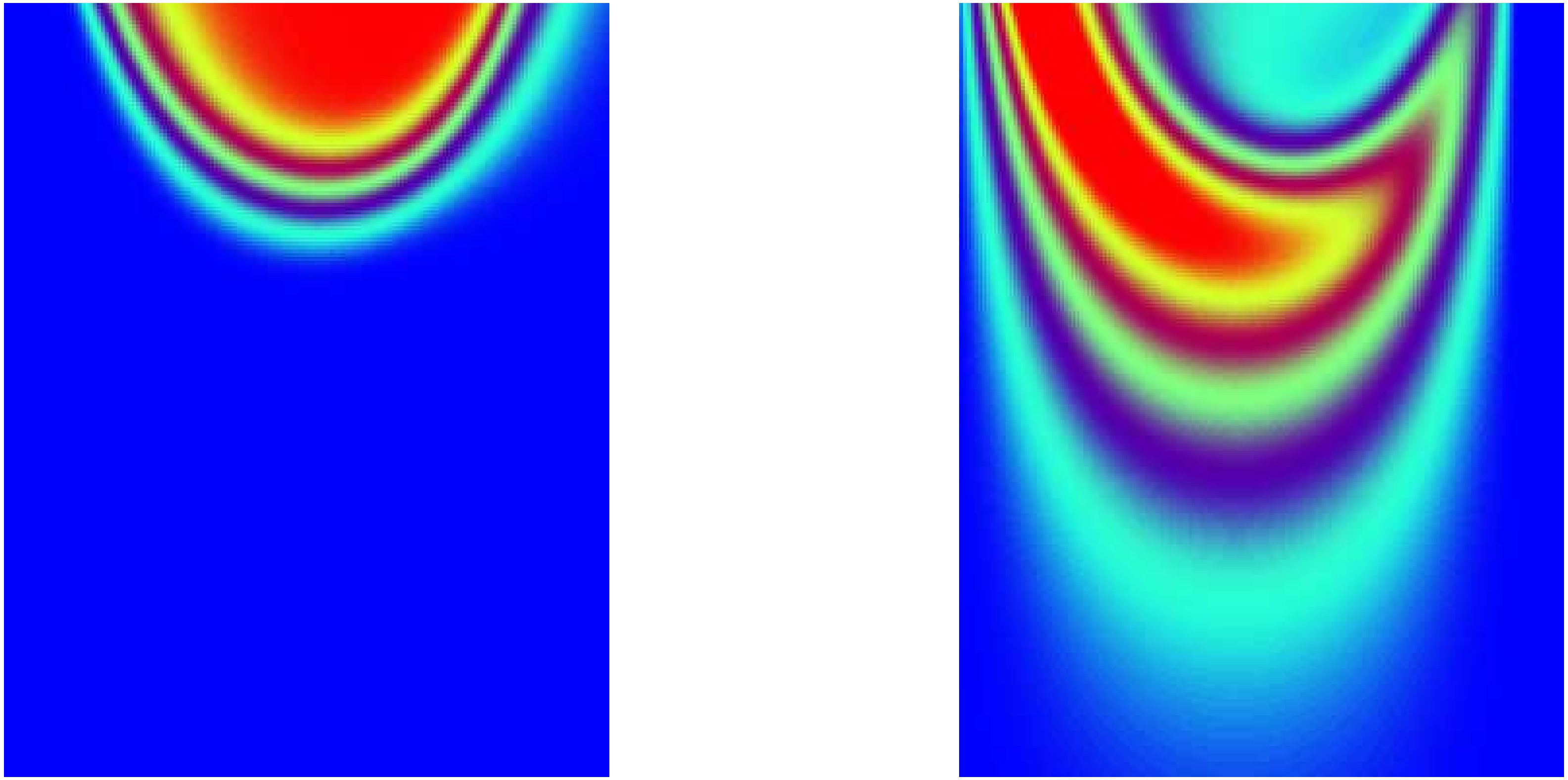
Photo adaptation

W = 0 m s^{-1}, turbidity: Jerlov I



Inhibition parameter

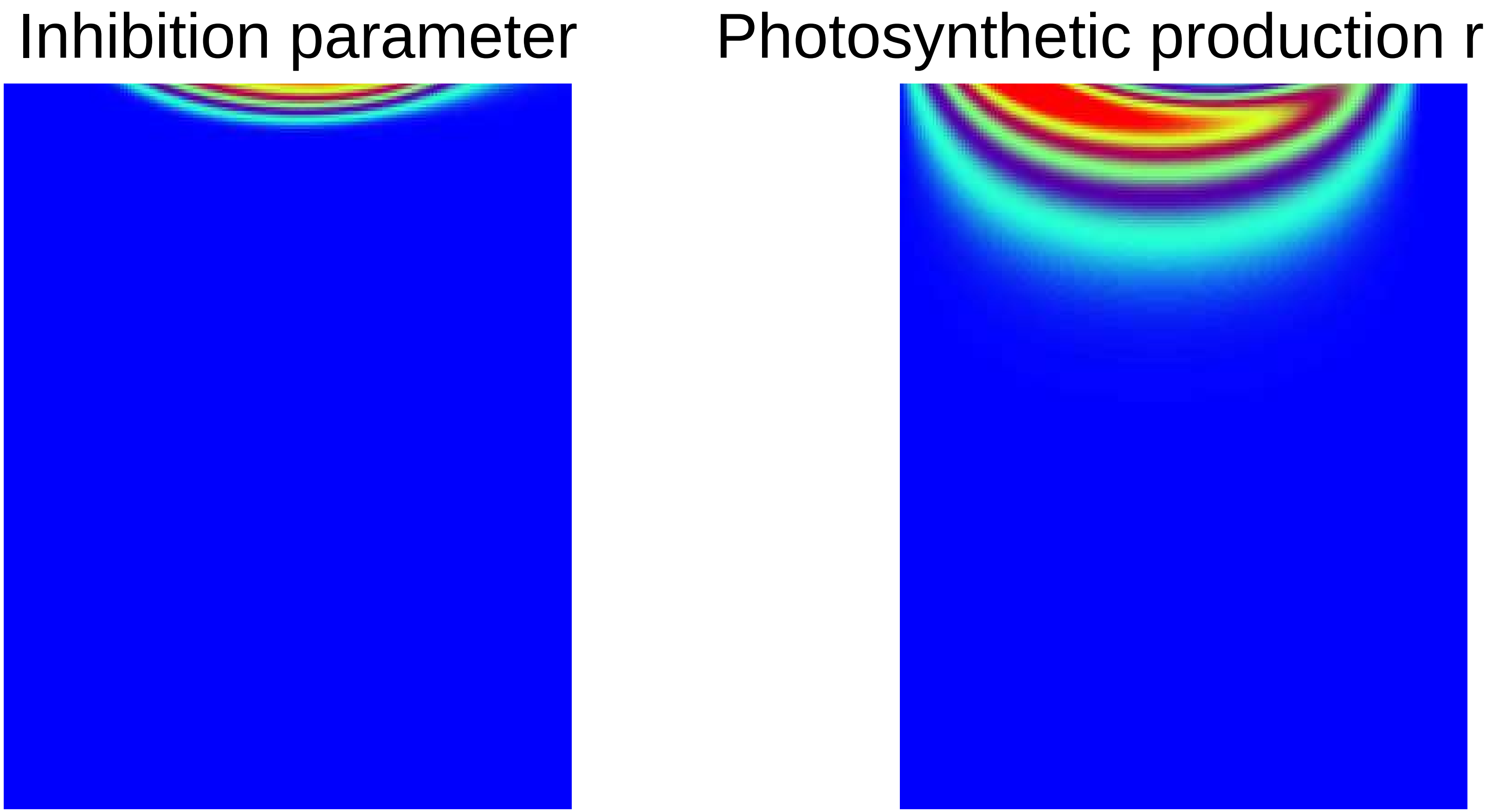
Photosynthetic production rate

x-axis: 6:00 h $\leq t \leq$ 19:00 h, z-axis: -100 m $\leq z \leq 0$ m

Contours left: 0 $\leq Y \leq 1$; right: 0 $\leq P \leq 0.004$ pg-at O_2 cell^{-1}h^{-1}

Photo adaptation

W = 0 m s^{-1}, turbidity: Jerlov III



Inhibition parameter

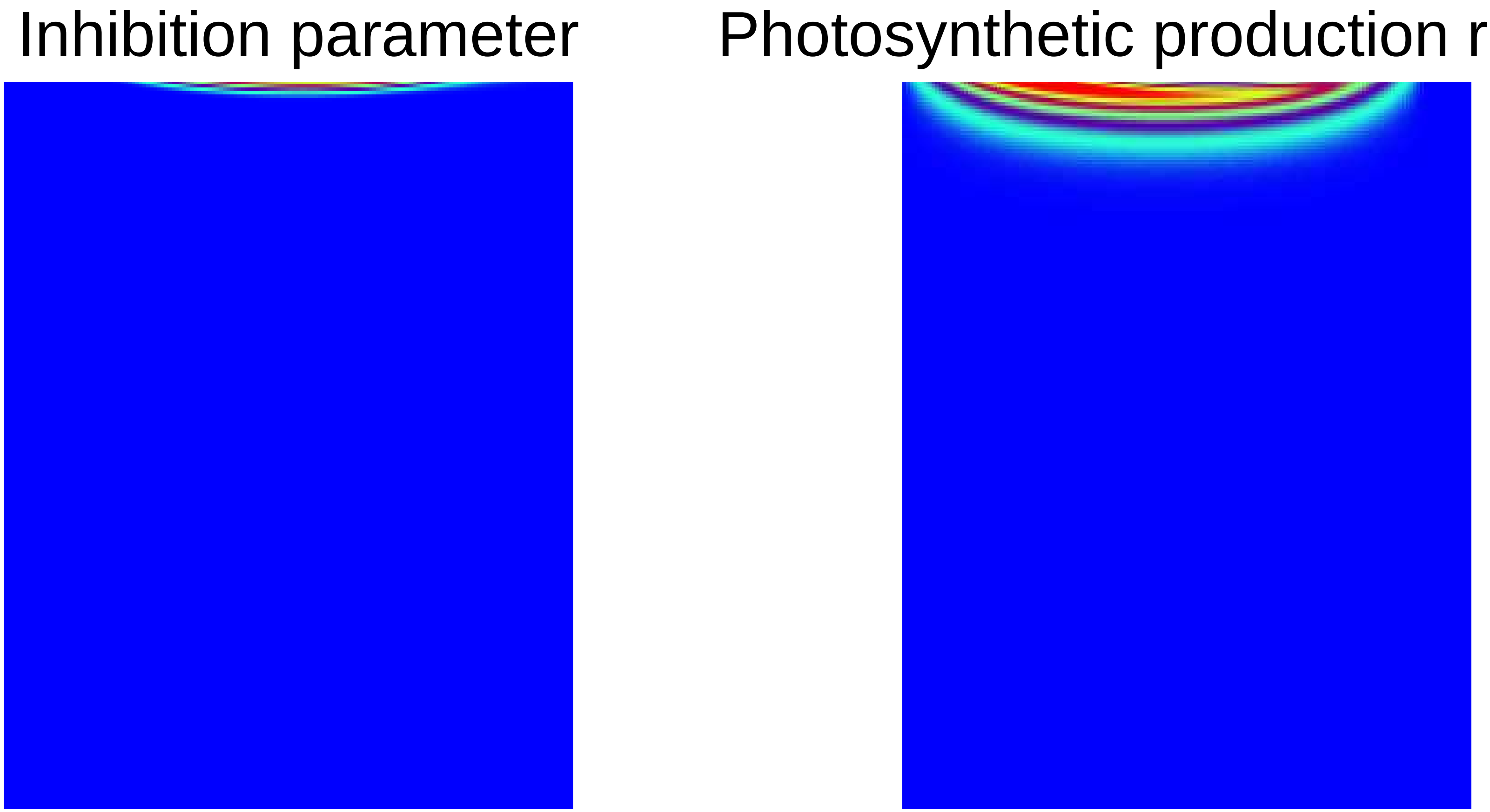
Photosynthetic production rate

x-axis: 6:00 h $\leq t \leq$ 19:00 h, z-axis: -100 m $\leq z \leq 0$ m

Contours left: 0 $\leq Y \leq 1$; right: 0 $\leq P \leq 0.004$ pg-at O_2 cell^{-1}h^{-1}

Photo adaptation

W = 0 m s^{-1}, turbidity: coastal water

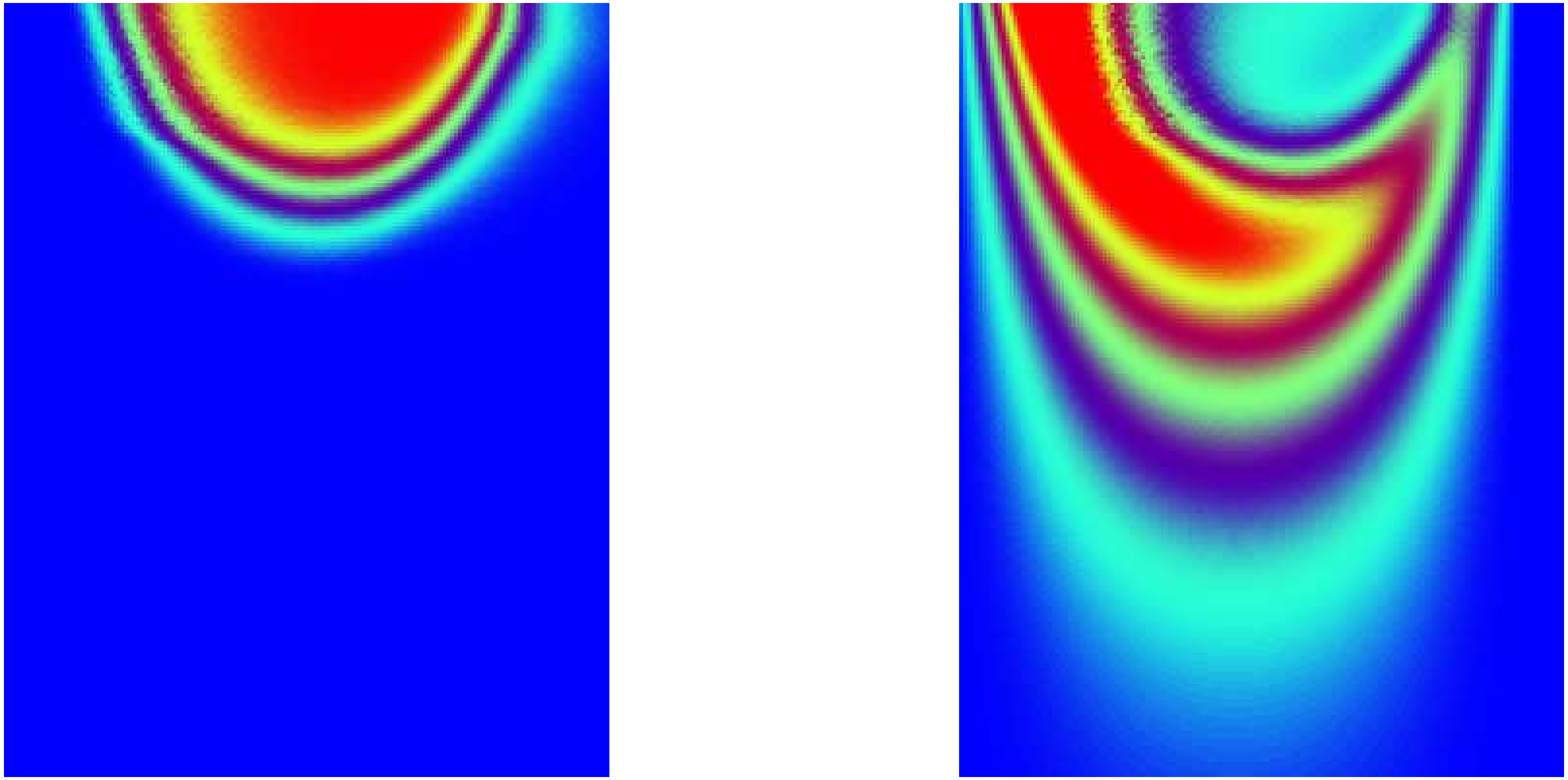


x-axis: 6:00 h $\leq t \leq$ 19:00 h, z-axis: -100 m $\leq z \leq 0$ m

Contours left: 0 $\leq Y \leq 1$; right: 0 $\leq P \leq 0.004$ pg-at O_2 cell^{-1}h^{-1}

Photo adaptation

W = 5 $\mathrm{m\,s^{-1}}$, turbidity: Jerlov I



Inhibition parameter

Photosynthetic production rate

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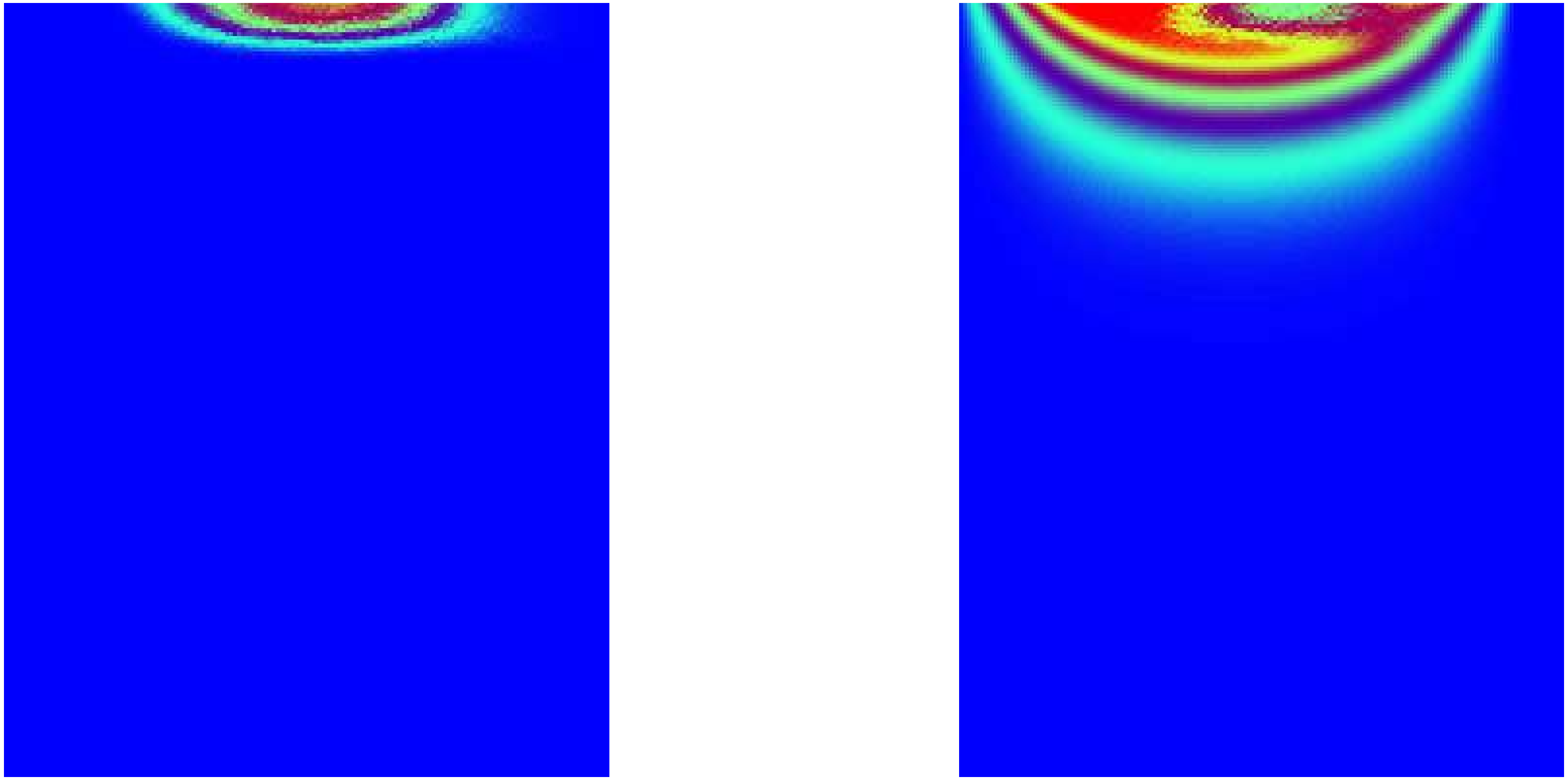
Photo adaptation

W = 5 m s^{-1}, turbidity: Jerlov III



Inhibition parameter

Photosynthetic production rate

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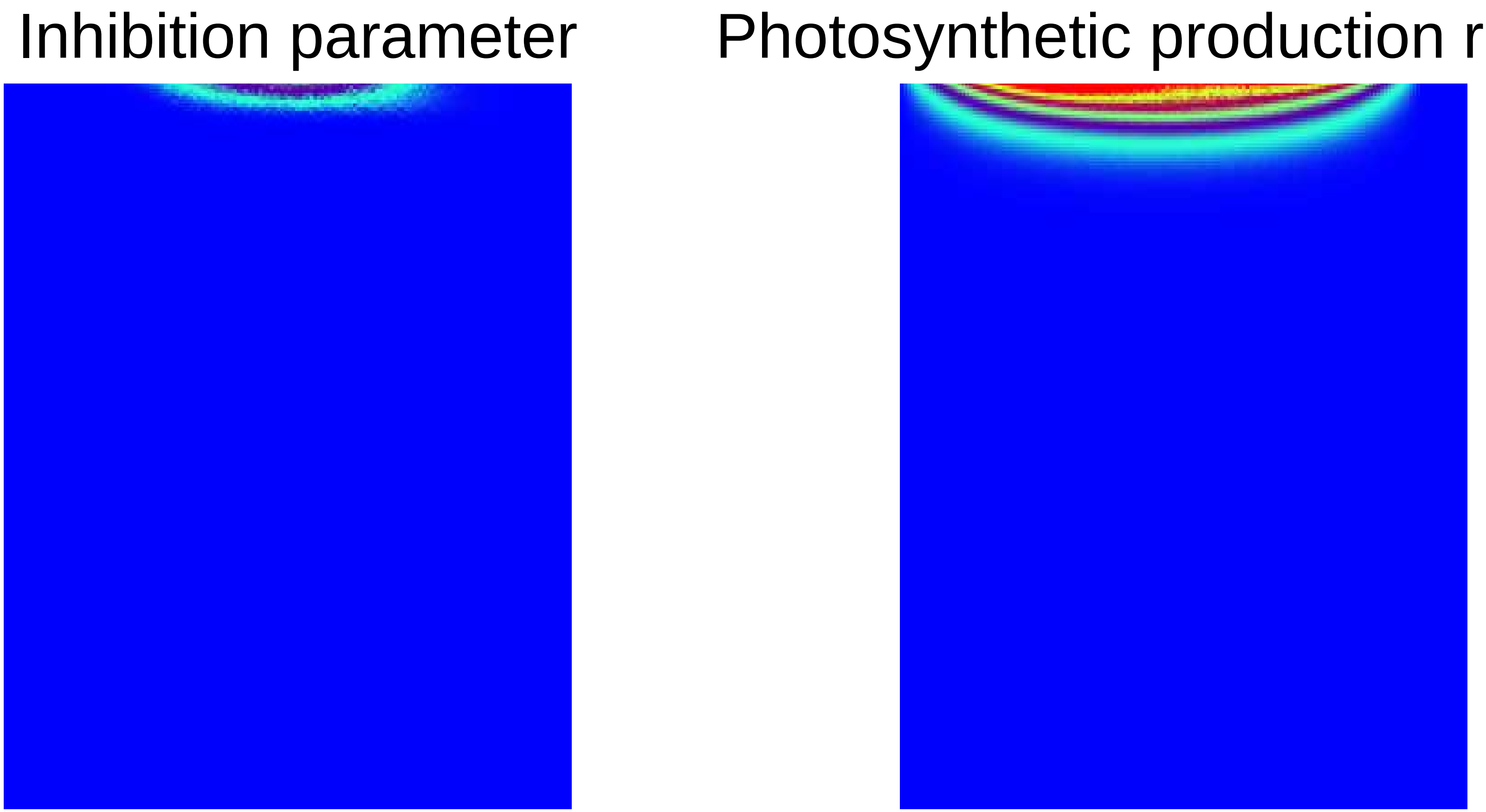
Photo adaptation

W = 5 m s^{-1}, turbidity: coastal water



Inhibition parameter

Photosynthetic production rate

x-axis: 6:00 h $\leq t \leq$ 19:00 h, z-axis: -100 m $\leq z \leq 0$ m

Contours left: 0 $\leq Y \leq 1$; right: 0 $\leq P \leq 0.004$ pg-at O_2 cell^{-1}h^{-1}

Photo adaptation

W = 10 m s^{-1}, turbidity: Jerlov I

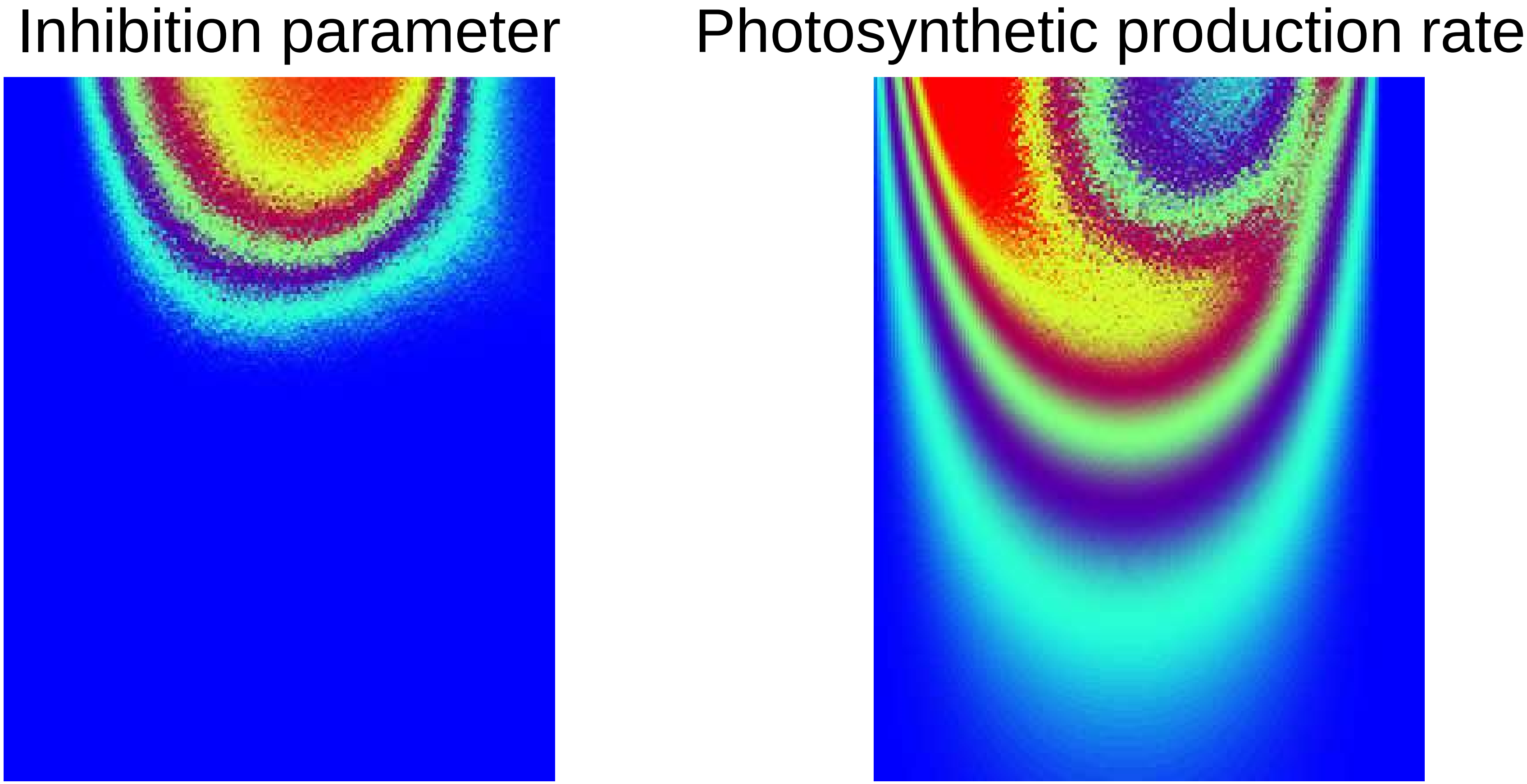


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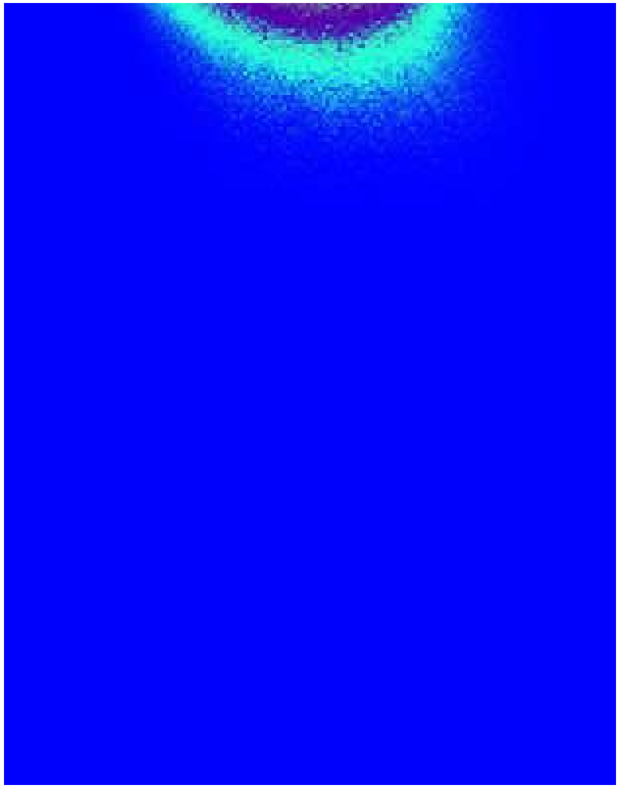
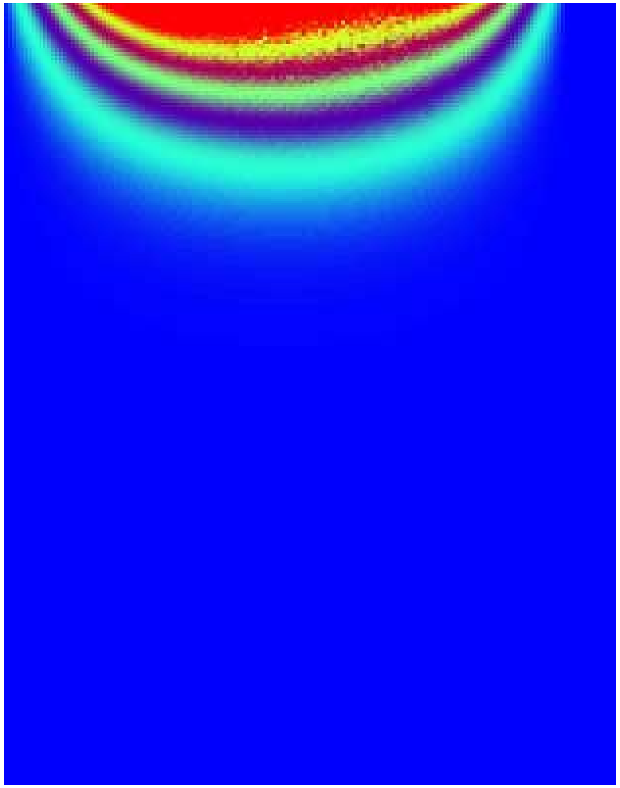
Photo adaptation

W = 10 m s^{-1}, turbidity: Jerlov III

Inhibition parameter



Photosynthetic production rate



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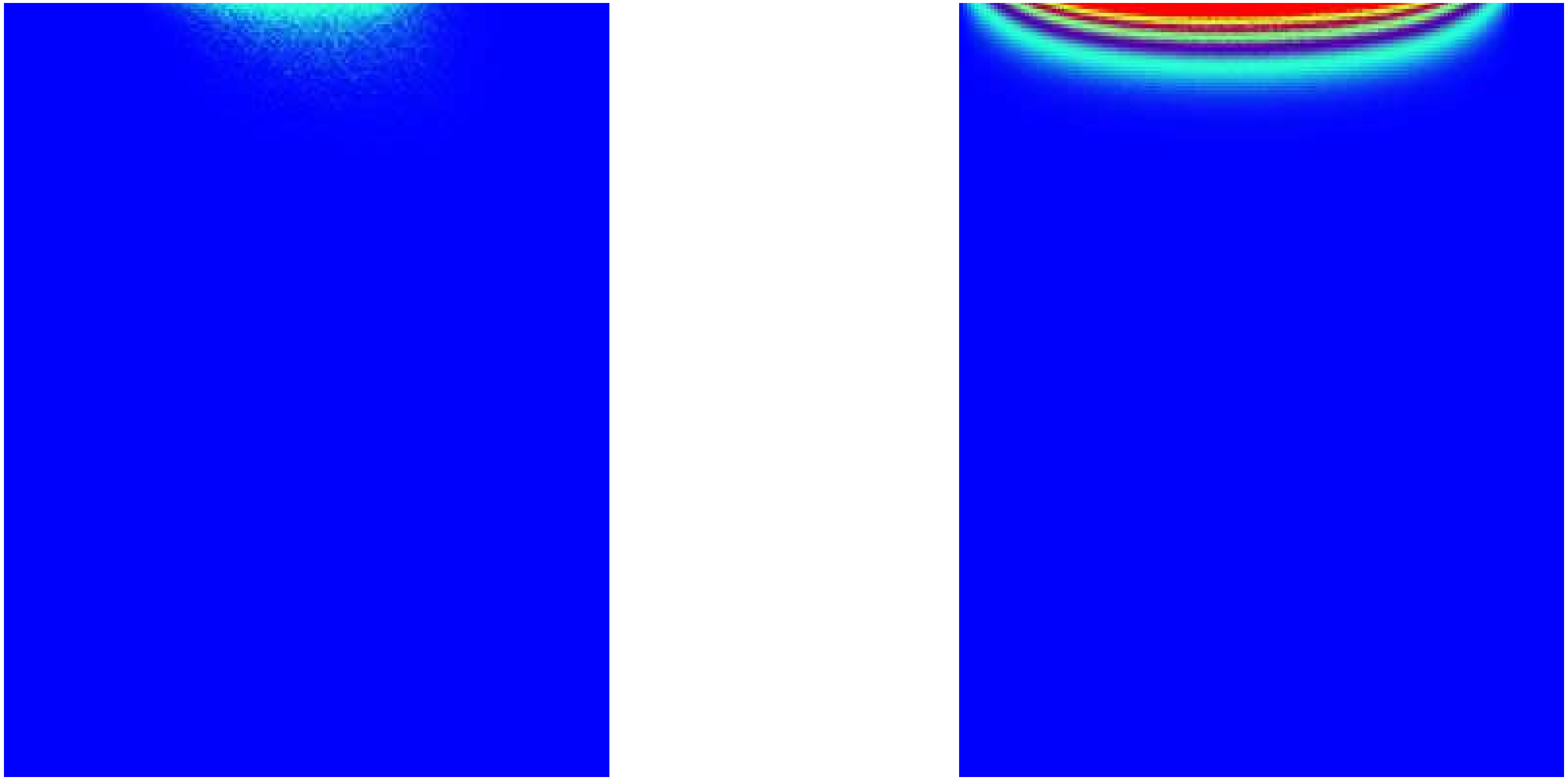
Contours left: 0 $\leq Y \leq 1$; right: 0 $\leq P \leq 0.004$ pg-at O_2 cell^{-1}h^{-1}

Photo adaptation

W = 10 m s^{-1}, turbidity: coastal water



Inhibition parameter

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