Parameterising the wave-current bottom boundary layer

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Contents

Problem

- Waves in shallow water have a strong impact on the BBL, but are not resolved in statistical turbulence closure models.
- There are empirical wave-current interaction models (Grant and Madsen), but they are not compatible with statistical turbulence closure models.
- Mellor (2002) suggested to parameterise the wave impact on the BBL by an extra turbulence production term.
- We are taking up the Mellor suggestion and propose to refine and generalise it.

Mellor's suggestion

The essence of Mellor (2002):

- 1D models can resolve the wave-current BBL, 3D models can't.
- The MY 1D model is tested successfully against lab data from Jensen et al. (1989).
- In order to parameterise the effect of waves on the BBL, an extra turbulence shear production P_A is included:

$$\left(\frac{P_A}{\omega u_b^2}\right)^{1/3} = F_\phi(\phi) F_z\left(\frac{z\omega}{u_b}, \frac{z_0\omega}{u_b}\right) \tag{1}$$

- The functions F_ϕ and F_z are then fitted to results from the 1D model.

Reynolds-averaged dynamic equations

$$\partial_{t'} u' - \partial_{z'} \left(K'_m \partial_{z'} u' \right) = -g \left(S_x + S_w f \left(2\pi \frac{t'}{T} \right) \right) \tag{2}$$

(3)

$$\partial_{t'} v' - \partial_{z'} \left(K'_m \partial_{z'} v' \right) = -g S_y \tag{4}$$

$g\rho_0 S_x$: mean pressure gradient in x-direction

$g\rho_0 S_y$: mean pressure gradient in y-direction

$g\rho_0 S_w$: max. wave-induced pressure gradient in x-direction

f is periodic, $\max|f| = 1$, $\int_0^T f\left(2\pi \frac{t}{T}\right)$ d$t = 0$.

Dimensionless form

$$\underline{S_i}\partial_t u - \partial_z \left(K_m \partial_z u\right) = -S_x^r - f\,(2\pi t) \tag{5}$$

(6)

$$\underline{S_i}\partial_t v - \partial_z \left(K_m \partial_z v\right) = -S_y^r \tag{7}$$

$$t = \frac{t'}{T}, \qquad z = \frac{z'}{z_0}, \qquad u = \frac{u'}{u^*}, \qquad v = \frac{v'}{u^*}, \qquad K_m = \frac{K'_m}{u^* z_0},$$

$$u^* = \sqrt{g z_0 S_w}, \quad S_i = \frac{H}{u^* T}, \quad S_x^r = \frac{S_x}{S_w}, \quad S_y^r = \frac{S_y}{S_w}. \tag{8}$$

Wave-averaged equations

Decomposition in mean and wave-induced fluctuation:

$$u = \bar{u} + \tilde{u}, \quad v = \bar{v} + \tilde{v}, \tag{9}$$

with

$$\bar{u}(z) = \int_0^1 u(z,t)\mathrm{d}t, \quad \bar{\tilde{u}} = 0. \tag{10}$$

Wave-averaging of the dimensionless equations gives:

$$\underline{S_i}\partial_t \bar{u} - \partial_z \overline{(K_m \partial_z u)} = -S_x^r \tag{11}$$

$$\tag{12}$$

$$\underline{S_i}\partial_t \bar{v} - \partial_z \overline{(K_m \partial_z v)} = -S_y^r \tag{13}$$

Wave-averaged equations (cont.)

Reformulation of the stresses:

$$\underline{S_i}\partial_t\bar{u} - \partial_z\left(\left(\bar{K}_m + \frac{\overline{\tilde{K}_m\partial_z\tilde{u}}}{\partial_z\bar{u}}\right)\partial_z\bar{u}\right) = -S_x^r \qquad (14)$$

$$(15)$$

$$\underline{S_i}\partial_t\bar{v} - \partial_z\left(\left(\bar{K}_m + \frac{\overline{\tilde{K}_m\partial_z\tilde{v}}}{\partial_z\bar{v}}\right)\partial_z\bar{v}\right) = -S_y^r \qquad (16)$$

Wave-averaged equations (cont.)

Mean kinetic energy equation:

$$
\underline{S_i}\partial_t\left(\frac{\bar u^2}{2}+\frac{\bar v^2}{2}\right)-\partial_z\left(\bar K_m\partial_z\left(\frac{\bar u^2}{2}+\frac{\bar v^2}{2}\right)\right)
$$

$$
-\partial_z\left(\frac{\overline{\tilde K_m\partial_z\tilde u}}{\partial_z\bar u}\partial_z\left(\frac{\bar u^2}{2}\right)\right)-\partial_z\left(\frac{\overline{\tilde K_m\partial_z\tilde v}}{\partial_z\bar v}\partial_z\left(\frac{\bar v^2}{2}\right)\right)=
$$

$$
-\underbrace{\left\{\bar K_m\left((\partial_z\bar u)^2+(\partial_z\bar v)^2\right)+\frac{\overline{\tilde K_m\partial_z\tilde u}}{\partial_z\bar u}(\partial_z\bar u)^2+\frac{\overline{\tilde K_m\partial_z\tilde v}}{\partial_z\bar v}(\partial_z\bar v)^2\right\}}_{\text{Total shear production } \bar P}
$$

$$
-S^r_x\bar u-S^r_y\bar v. \tag{17}
$$

Strategy

- Run GOTM for relevant range of S_i, S_x^r, S_y^r, H/z_0 (e.g. 10^4 model runs, until periodic steady state).
- Compute wave stresses $\overline{\tilde{K}_m \partial_z \tilde{u}}$ and $\overline{\tilde{K}_m \partial_z \tilde{v}}$.
- Find analytical approximations to these stresses (either as functions of z/z_0 or integrated (for parameterisation via bottom TKE fluxes).
- Implement analytical approximations as lookup table into GOTM.
- Assess accuracy of the empirical scheme(s).
- Implement new parameterisation into 3D models: additional problem of wave-induced stress tensor divergence, see e.g. $\partial_x \overline{\tilde{u}^2}$, $\partial_x \overline{\tilde{C}\tilde{u}}$ (Davies and Villaret, 2002).

Algebraic SMCs

Turbulent Fluxes:

$$\langle u'w' \rangle = -K_m \partial_z \langle u \rangle, \quad \langle w'\theta' \rangle = -K_h \partial_z \langle \theta \rangle$$

Eddy Viscosity / Eddy Diffusivity:

$$K_m = c_\mu(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon}, \quad K_h = c'_\mu(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon}.$$

Shear Number, Buoyancy Number:

$$\alpha_M = \frac{k^2}{\varepsilon^2} M^2, \quad \alpha_N = \frac{k^2}{\varepsilon^2} N^2.$$

Statistical turbulence modelling

TKE equation:

$$\partial_t k - \partial_z \left(\frac{K_m}{\sigma_k} \partial_z k \right) = P + G - \varepsilon \tag{18}$$

Dissipation rate equation:

$$\partial_t \varepsilon - \partial_z \left(\frac{K_m}{\sigma_\varepsilon} \partial_z \varepsilon \right) = \frac{\varepsilon}{k} \left(c_{\varepsilon 1} P + c_{\varepsilon 3} B - c_{\varepsilon 2} \varepsilon \right) \tag{19}$$

Production terms:

$$P = K_m \left((\partial_z u)^2 + (\partial_z v)^2 \right) = K_m M^2, \quad B = K_h \partial_z b = K_h N^2 \tag{20}$$

Statistical turbulence modelling

kL equation (*Mellor and Yamada* [1982]):

$$\partial_t (kL) - \partial_z \left(S_l \partial_z (kL)\right) = \frac{L}{2}\left[E_1 P + E_3 B - \left(1 + E_2 \left(\frac{L}{L_z}\right)^2\right)\varepsilon\right]. \quad (21)$$

Generic length scale equation (*Umlauf and Burchard* [2003]):

$$\partial_t (k^n \varepsilon^m) - \partial_z \left(\frac{K_m}{\sigma_{nm}} k^n \varepsilon^m\right) = k^{n-1}\varepsilon^m \left(c_{nm1} P + c_{nm3} B - c_{nm2}\varepsilon\right). \quad (22)$$

This works without correction term only for $m \geq 0$.

NEW!

Dec. 15, 2005:
Release of GOTM 3.2.2

Dec. 08, 2005:
New plotting release

Mixing in lakes:
Do you know FLake?

Online: 5
Today: 24
Yesterday: 101
Total: 16535

About GOTM

General Ocean Turbulence Model (GOTM) is an ambitious name for a one-dimensional water column model for marine and limnological applications. It is coupled to a choice of traditional as well as state-of-the-art parameterisations for vertical turbulent mixing. The package consists of the FORTRAN90 software, a number of idealised and realistic test cases, and a scientific documentation, all published under the GNU public license. GOTM is, of course, not complete (and will never be). Whoever is missing any feature in GOTM, is warmly invited to contribute to GOTM and add her or his personal preferences or needs.

The idea behind GOTM is to provide a simple and well-documented tool that can be used to

- learn about the physics and numerical treatment of vertical mixing processes in natural waters by using a suite of prepared test cases,
- compare the performances of various turbulence schemes in different oceanic regimes (or compare them to known analytical solutions),
- integrate your own oceanic data into GOTM, and simulate numerically the water column processes under investigation,
- easily introduce new turbulence parameterisations into GOTM's modular structure,
- couple GOTM with new modules like air-sea interaction models, biological models, ice models, ... and investigate their behaviour in idealised or realistic test cases,
- and couple GOTM to your 3-D circulation model via a purpose-built interface that is applicable to all structured vertical z-level and topography-following grids (a number of 3-D models are already supported).

If you plan an extension to GOTM for your own research project, please contact us before you start. It occured a number of times in the past that different groups started working on very similar projects without knowing from each other - which is exactly against the spirit of the GOTM project.

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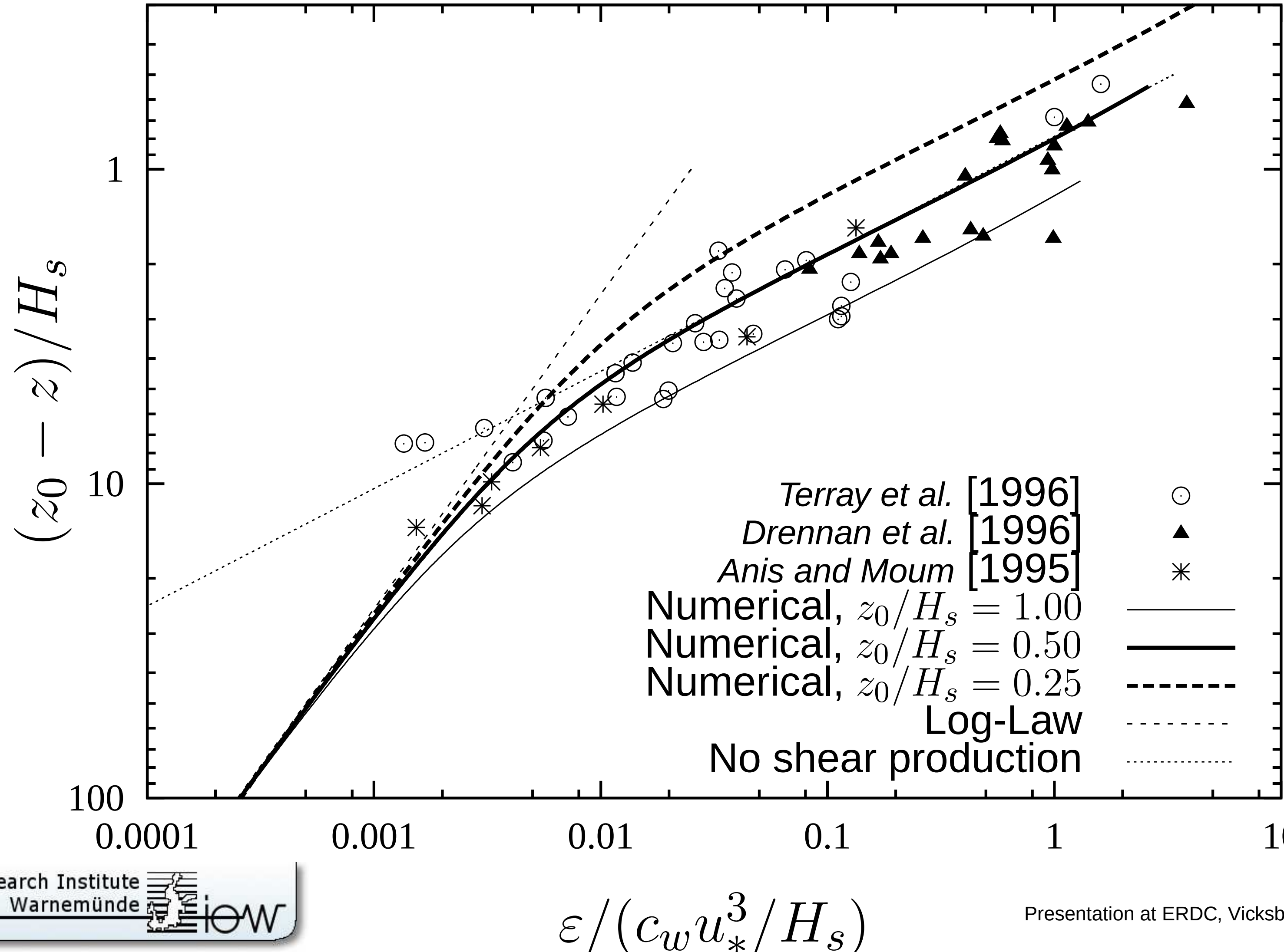
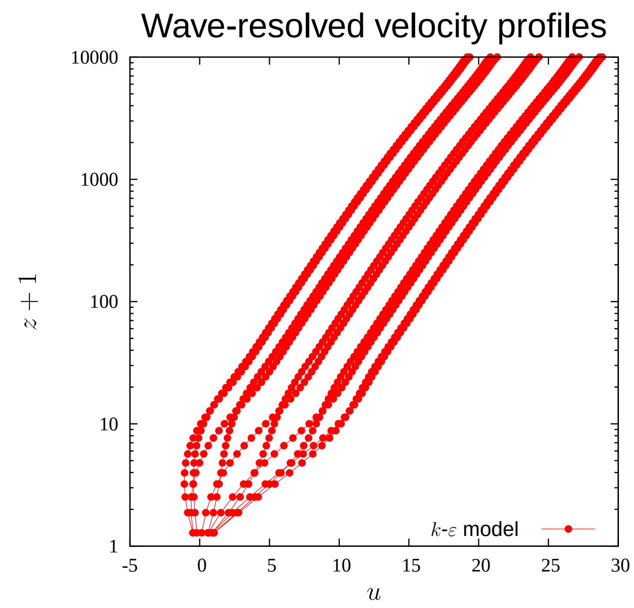
Surface wave breaking with GOTM

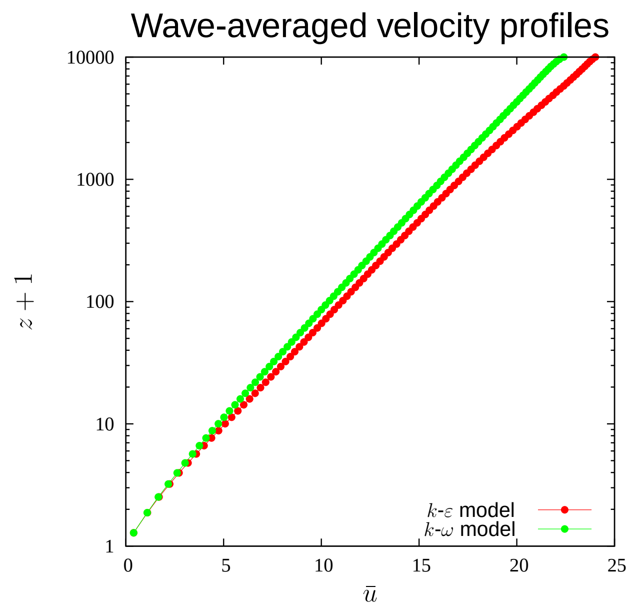
Observations and simulations of turbulent dissipation rate in the wave enhanced layer (from Burchard, 2001):



Wave-current BBL with GOTM

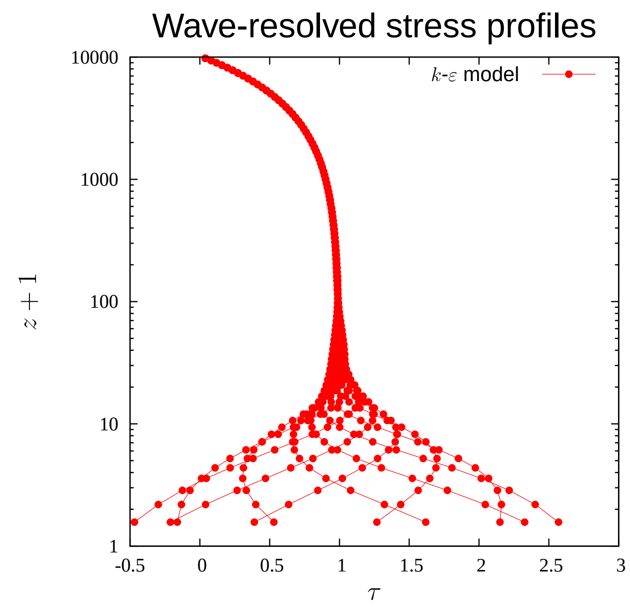
Example for wave-current BBL with GOTM, comparing the k-ε and the k-ω models:

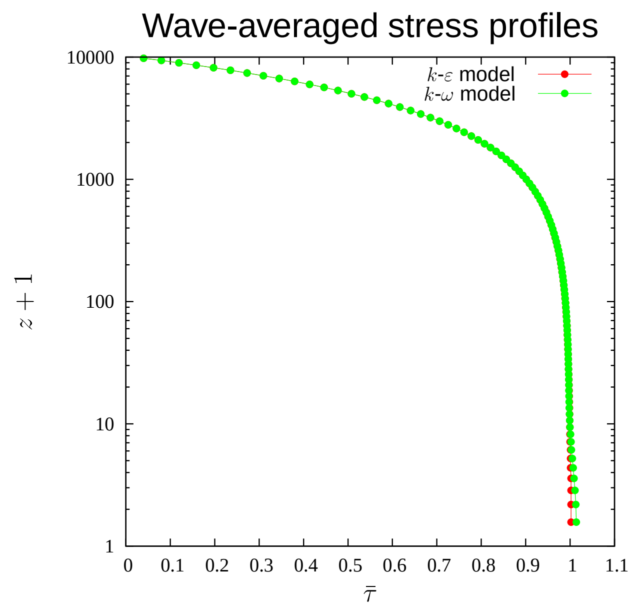




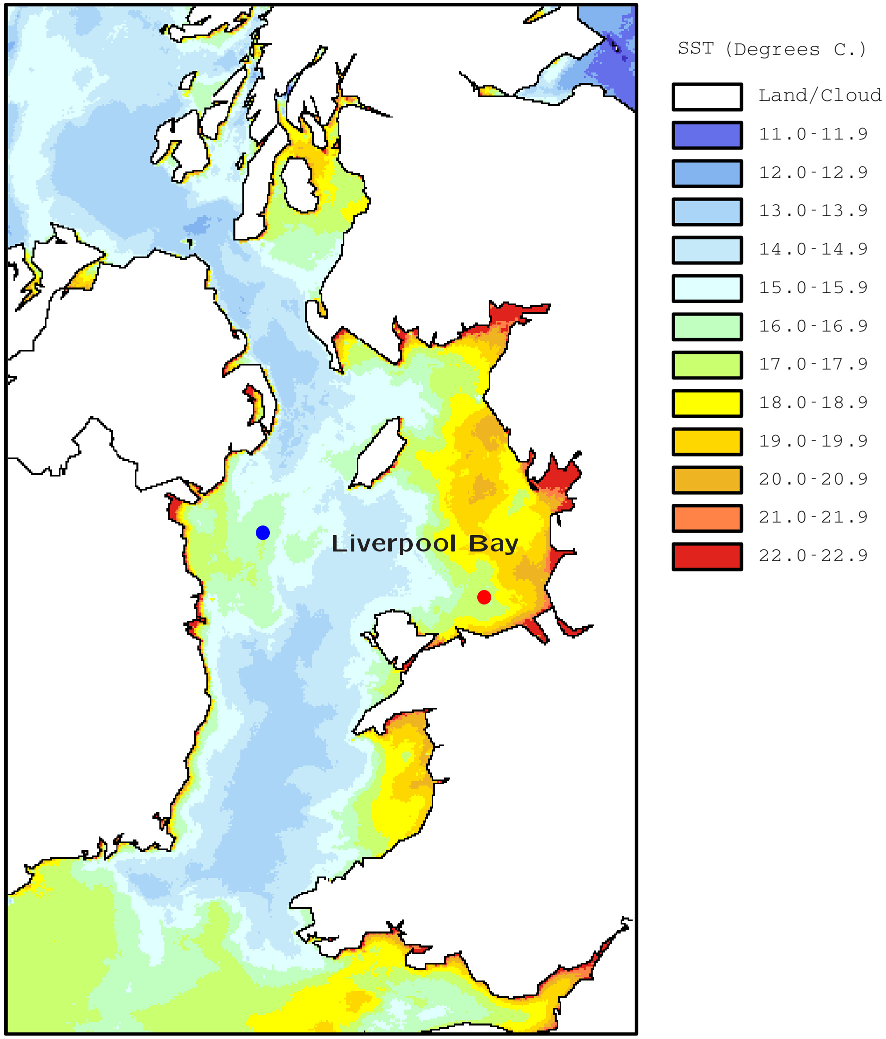
Wave-current BBL with GOTM

Example for wave-current BBL with GOTM, comparing the k-ε and the k-ω models:





Liverpool Bay



AVHRR Sea Surface Temperature image (11 July 1999, 17:00)

Baltic Sea Research Institute
Warnemünde

Pers. comm. Caren E. Binding

Liverpool Bay

Section of Temperature and Salinity

Fig 2.a Temperature (Degrees C)

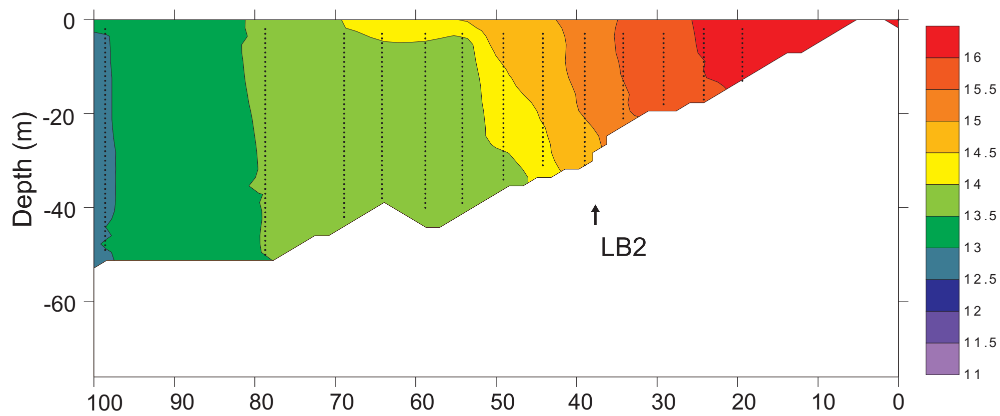
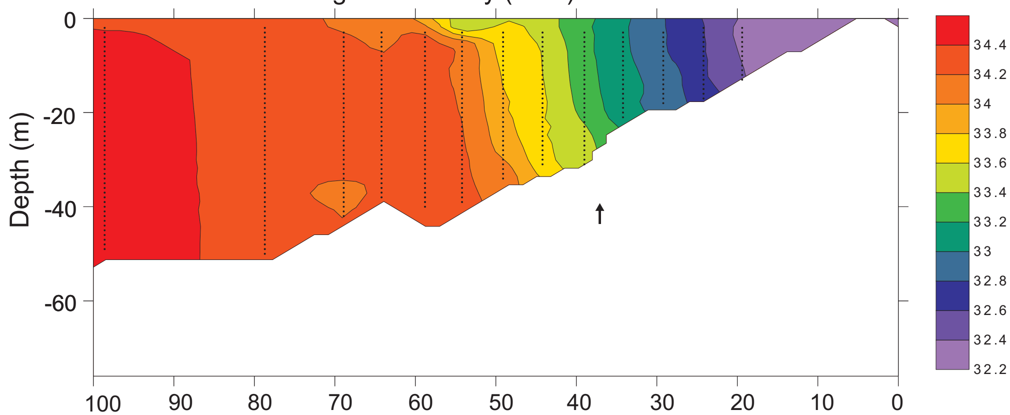


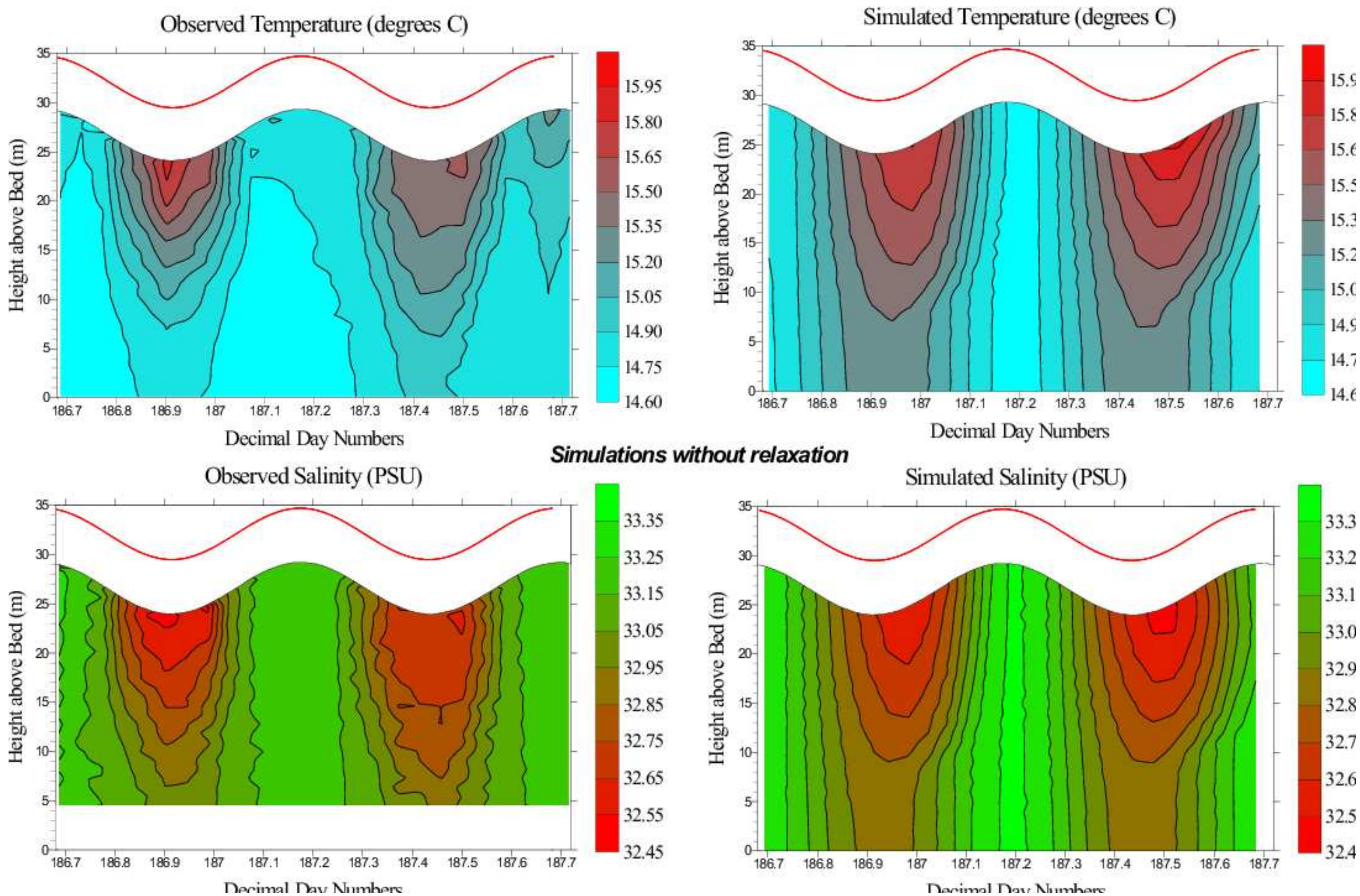
Fig 2.b Salinity (PSU)



Rippeth, Fisher, Simpson [2001]

Liverpool Bay

Observed and simulated temperature and salinity

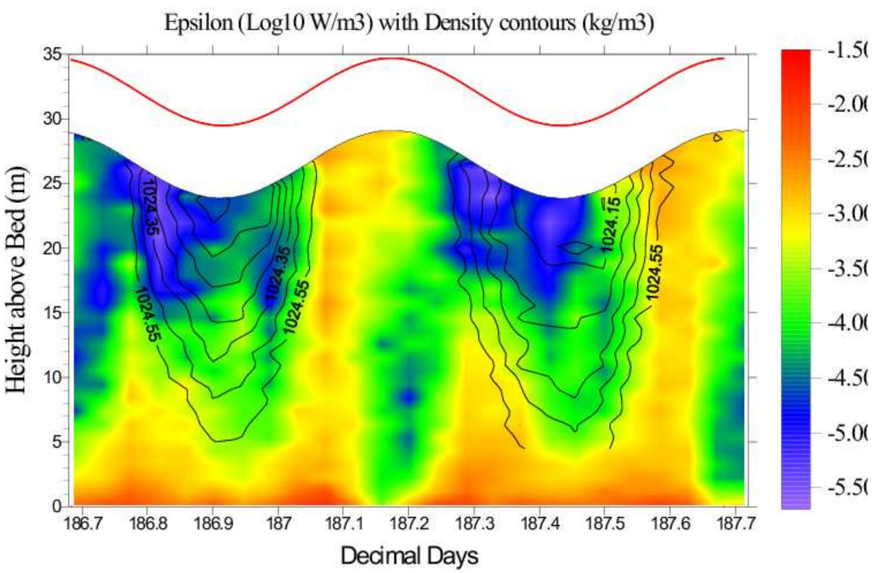


Simpson, Burchard, Fisher, Rippeth [2002]

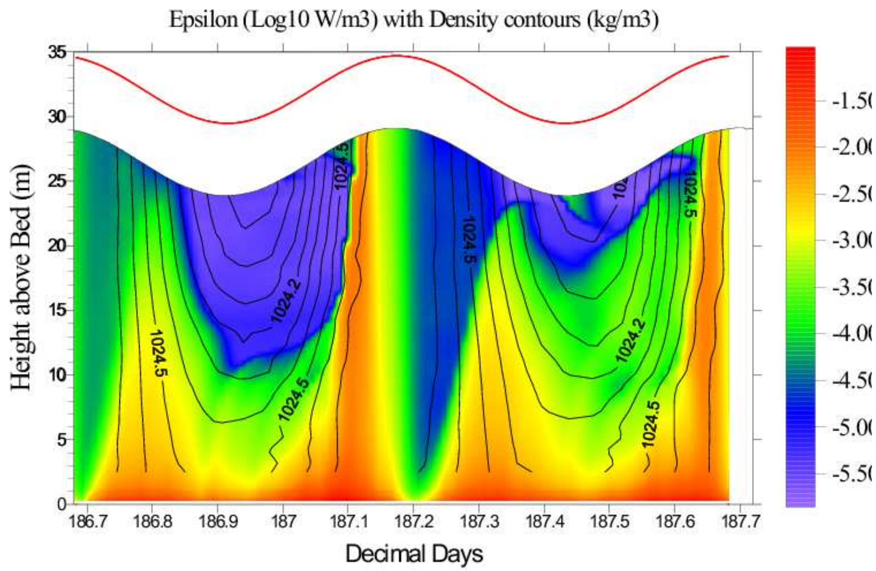
Liverpool Bay

Observed and simulated dissipation rates

Observations



CANUTO k-e (Nudge)



Simpson, Burchard, Fisher, Rippeth [2002]