

**Observations and numerical modelling  
of mixed layer turbulence:  
Do they represent the same statistical quantities ?**

**Hans Burchard and Hans Ulrich Lass**

`hans.burchard@io-warnemuende.de, uli.lass@io-warnemuende.de`

**Baltic Sea Research Institute Warnemünde**

# Or, with Steve Thorpe:

**Does it matter that ocean turbulence  
is neither homogeneous nor isotropic?**

# Reynolds decomposition

$$U = \langle U \rangle + \tilde{U}$$

$\langle U \rangle$  : **Mean part**

$\tilde{U}$  : **Fluctuating part**

**Averaging requirement:**

$$\langle \tilde{U} \rangle = 0.$$

# Averaging procedures

1. Linearity:

$$\langle U + \lambda V \rangle = \langle U \rangle + \lambda \langle V \rangle.$$

2. Exchange of Derivative and Averaging:

$$\langle \partial_x U \rangle = \partial_x \langle U \rangle$$

3. Double Averaging:

$$\langle \langle U \rangle \rangle = \langle U \rangle$$

4. Products of Averages:

$$\langle U \langle V \rangle \rangle = \langle U \rangle \langle V \rangle$$

# Averaging procedures

## Spatial average:

$$\langle U(\mathbf{x}, t) \rangle = \frac{1}{|V|} \iiint_V U(\boldsymbol{\xi}, t) d\boldsymbol{\xi}$$

## Temporal average:

$$\langle U(\mathbf{x}, t) \rangle = \frac{1}{|T|} \int_{t-T/2}^{t+T/2} U(x, \tau) d\tau$$

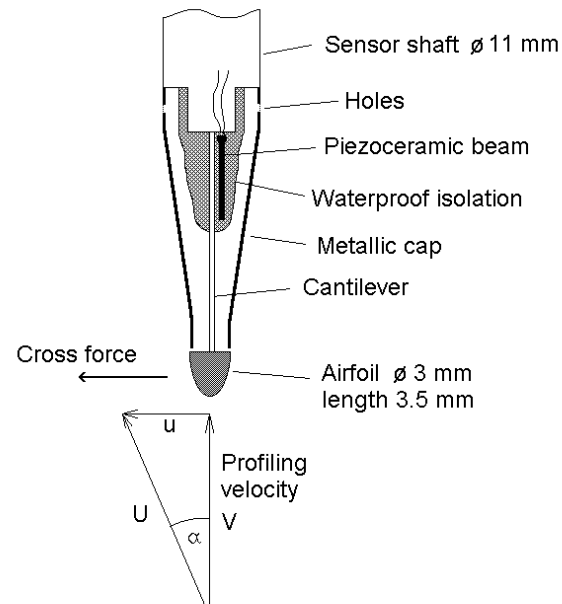
## Ensemble average:

$$\langle U(\mathbf{x}, t) \rangle = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n U_i(\mathbf{x}, t)$$

# About averaging procedures

- Only the ensemble averaging procedure is well defined in terms of theoretical considerations.
- Only for **homogeneous and stationary** turbulence, all averaging procedures are equivalent.

# Microstructure observations



- Microstructure observations are usually carried out by means of one-dimensional profiling techniques
- Three-dimensional turbulence is then quantified under the assumption of **local isotropy**.

# Bill Clinton:

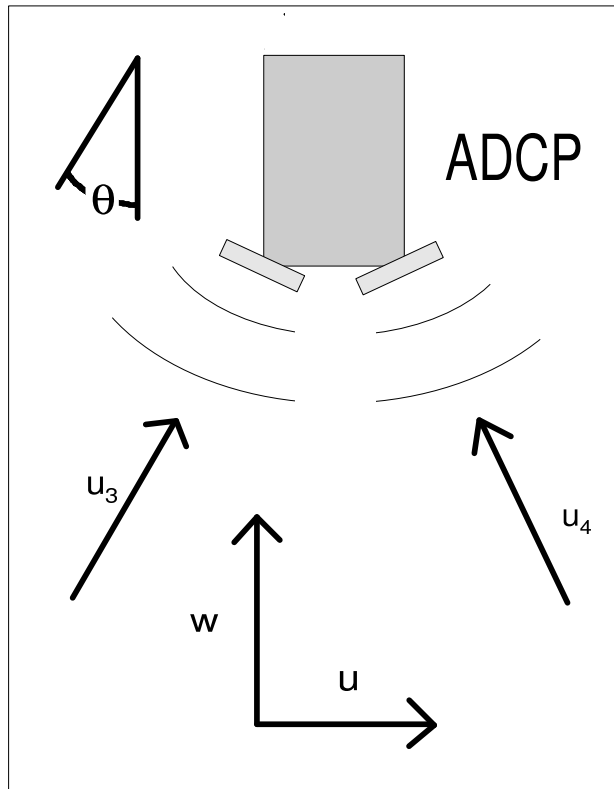
**It does matter that ocean turbulence  
is neither homogeneous (nor stationary)  
nor isotropic, stupid !**

# Heavy instrumentation helps



Source: <http://www.me.jhu.edu/~lefd/spiv/>

# High-resolution ADCP



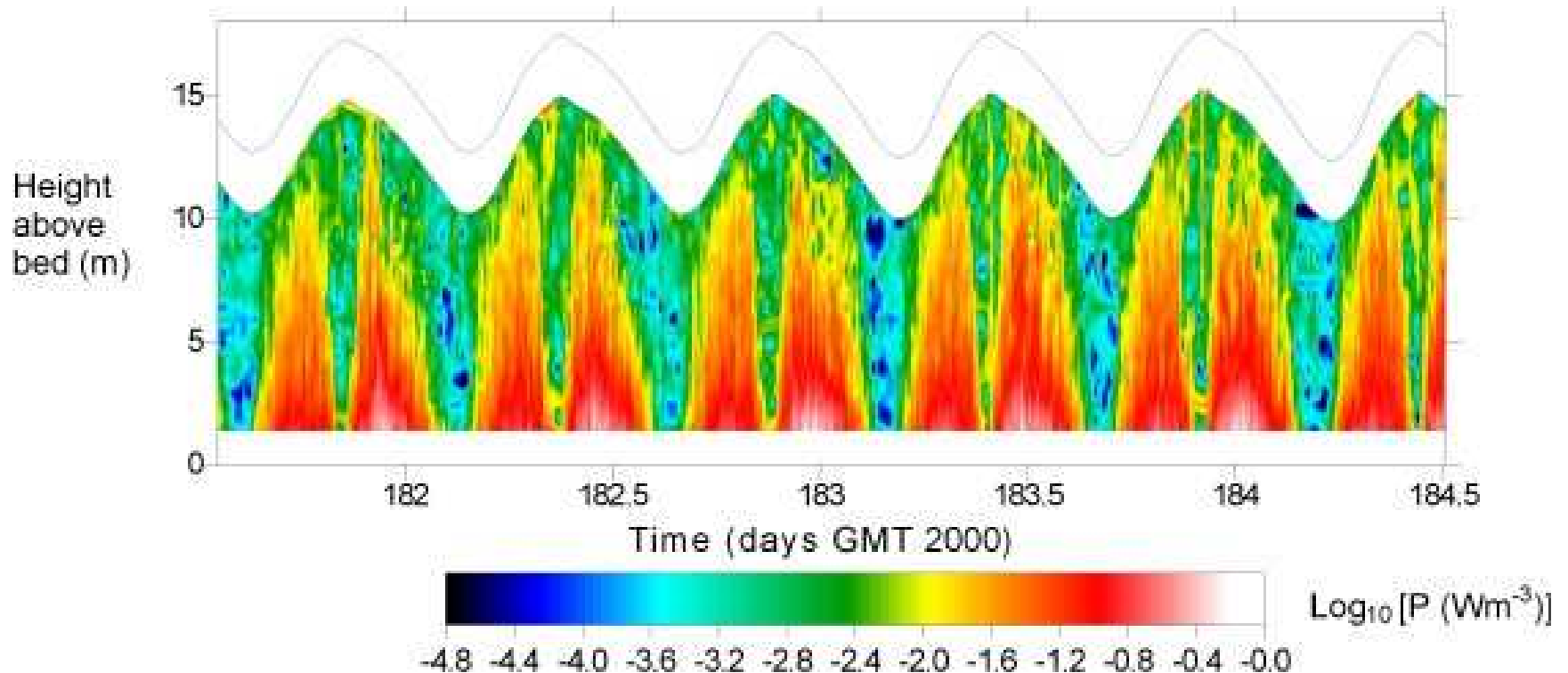
$$u_3 = u \sin \theta + w \cos \theta; \quad u_4 = u \sin \theta - w \cos \theta; \quad (1)$$

$$\begin{aligned} \langle \tilde{u}_3^2 \rangle &= \langle \tilde{u}^2 \rangle \sin^2 \theta + \langle \tilde{w}^2 \rangle \cos^2 \theta + 2 \langle \tilde{u} \tilde{w} \rangle \sin \theta \cos \theta \\ \langle \tilde{u}_4^2 \rangle &= \langle \tilde{u}^2 \rangle \sin^2 \theta + \langle \tilde{w}^2 \rangle \cos^2 \theta - 2 \langle \tilde{u} \tilde{w} \rangle \sin \theta \cos \theta \end{aligned} \quad (2)$$

$$\langle \tilde{u} \tilde{w} \rangle = \frac{\langle \tilde{u}_3^2 \rangle - \langle \tilde{u}_4^2 \rangle}{4 \sin \theta \cos \theta} \quad (3)$$

# Observing shear production

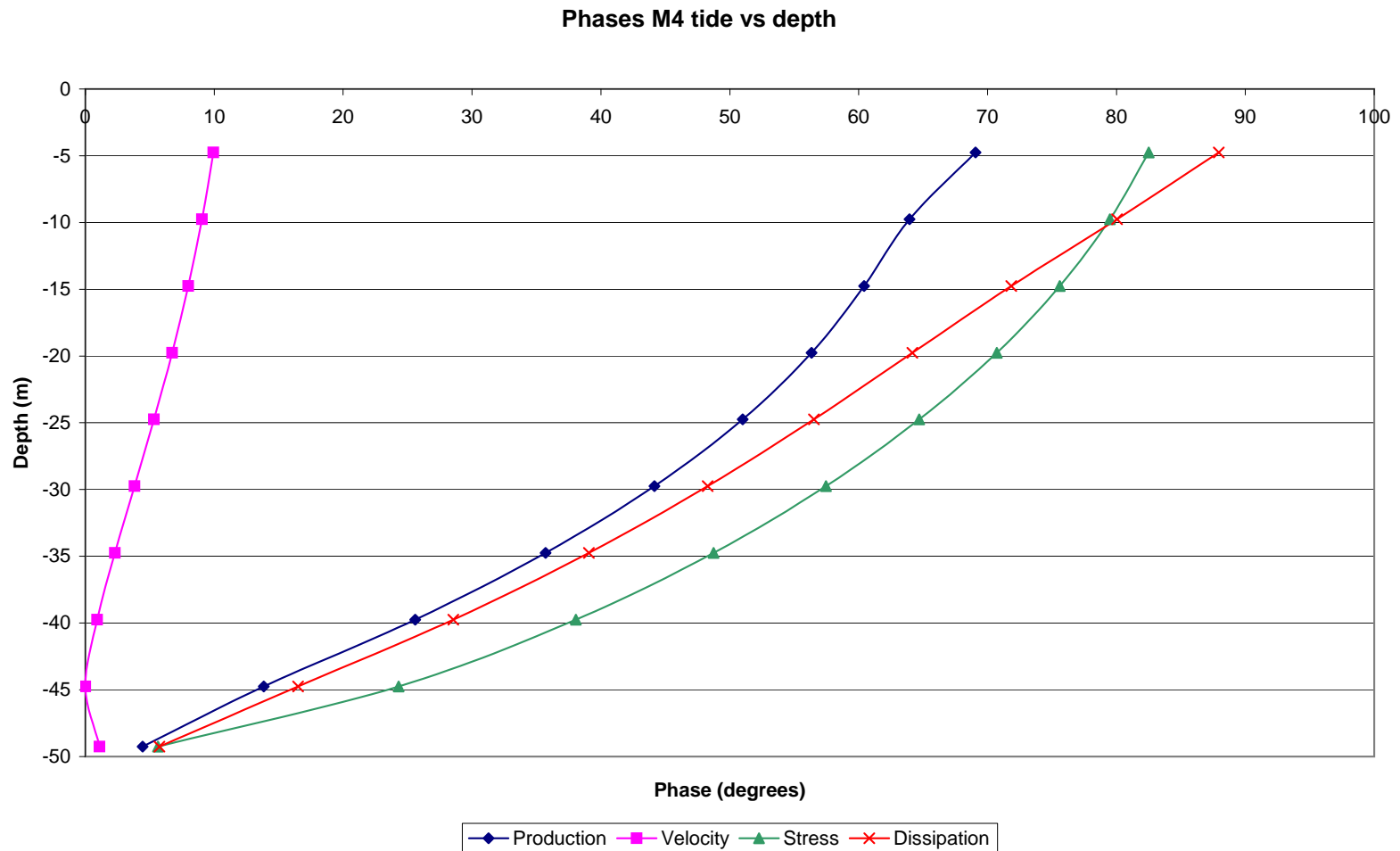
Example from Menai Strait, North Wales:



pers. comm. Tom P. Rippeth, Bangor, Wales

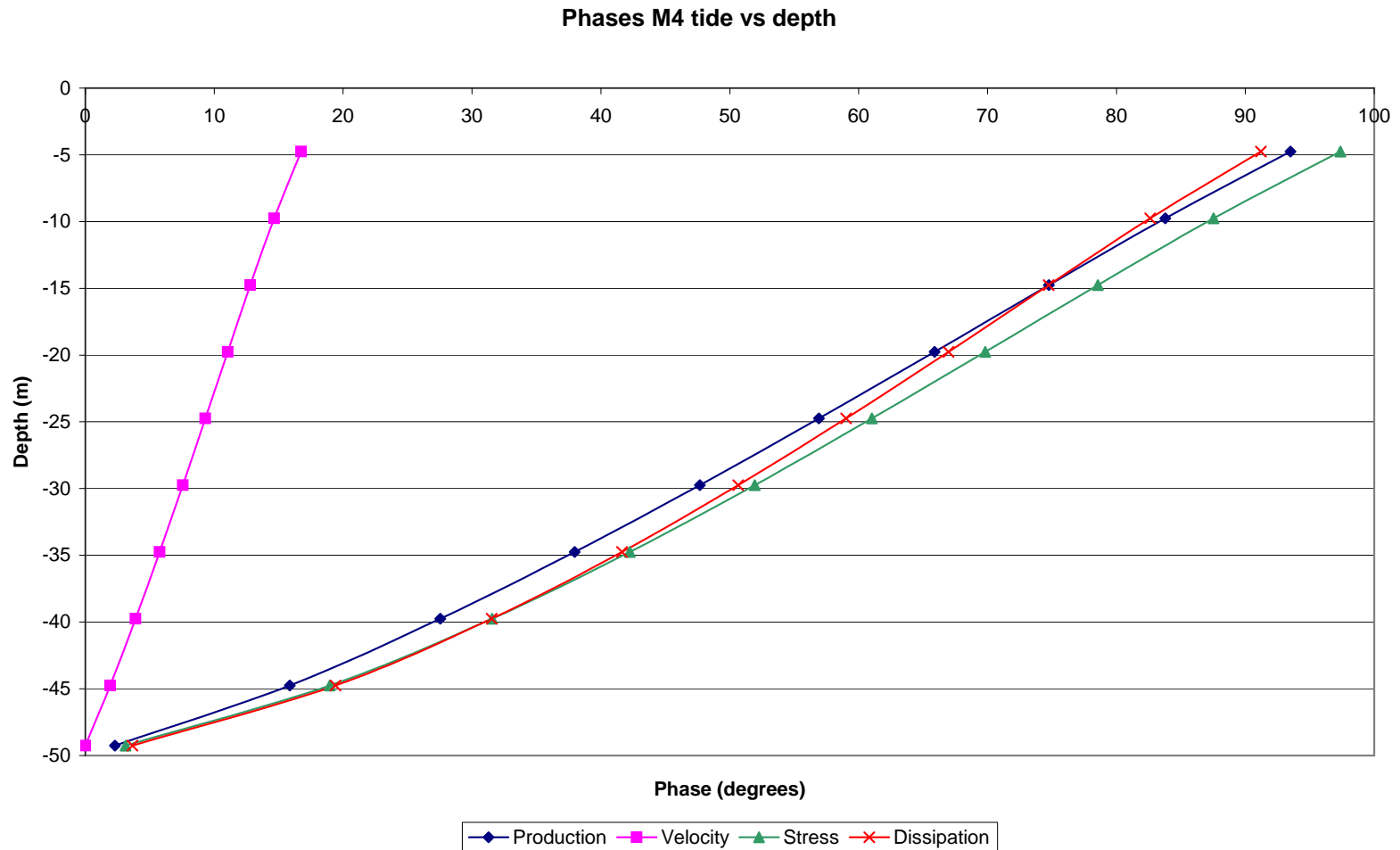
# Modelling tidal flow

## Two-equation turbulence model:



# Modelling tidal flow

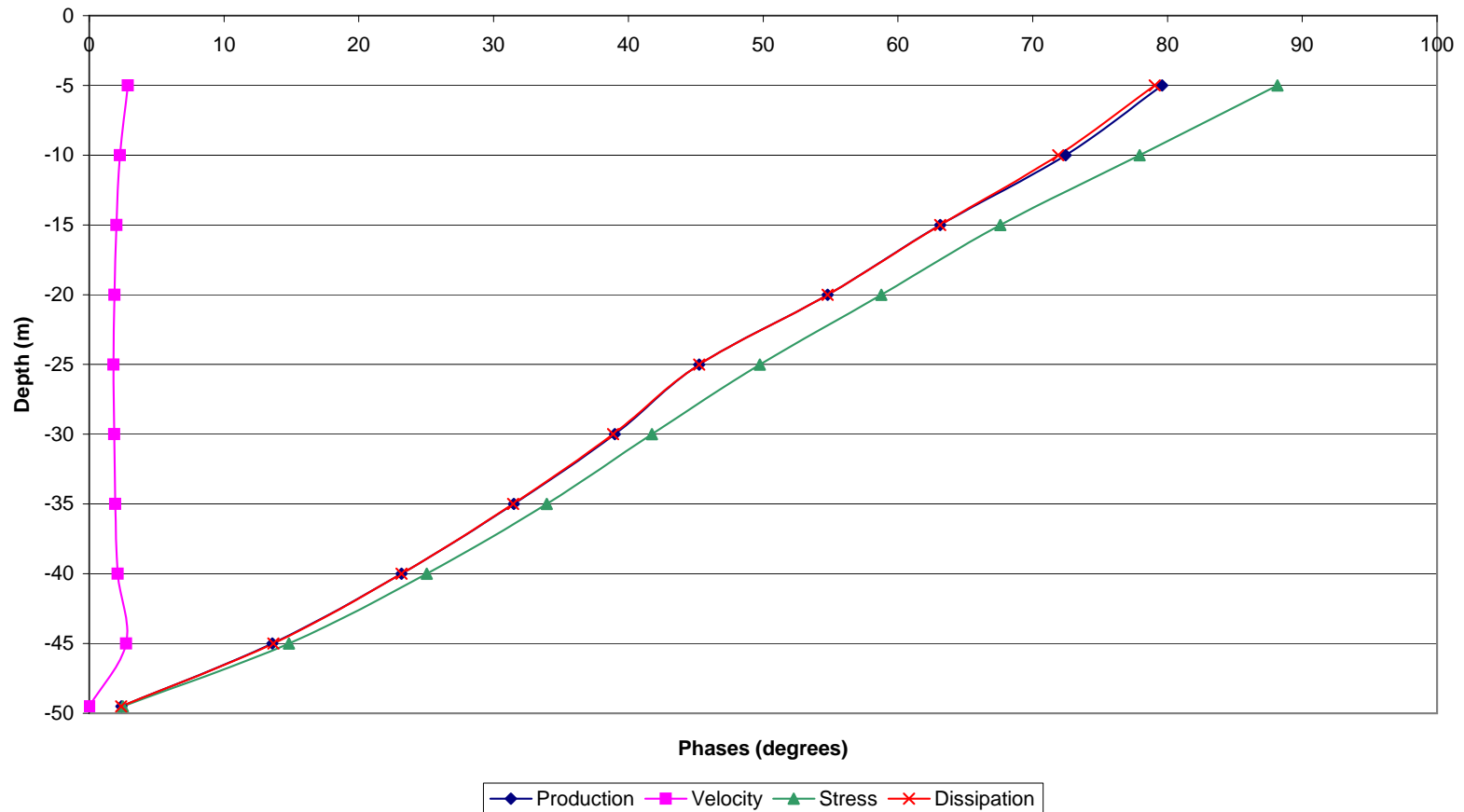
## One-equation turbulence model:



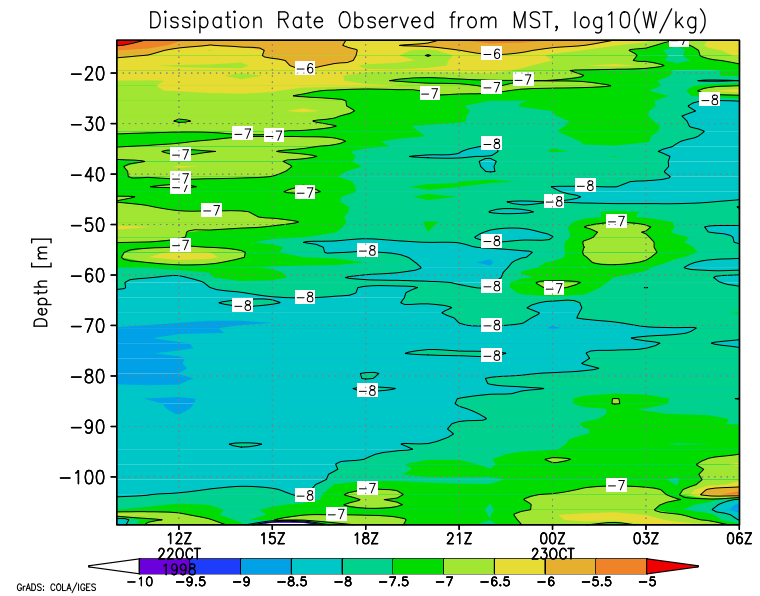
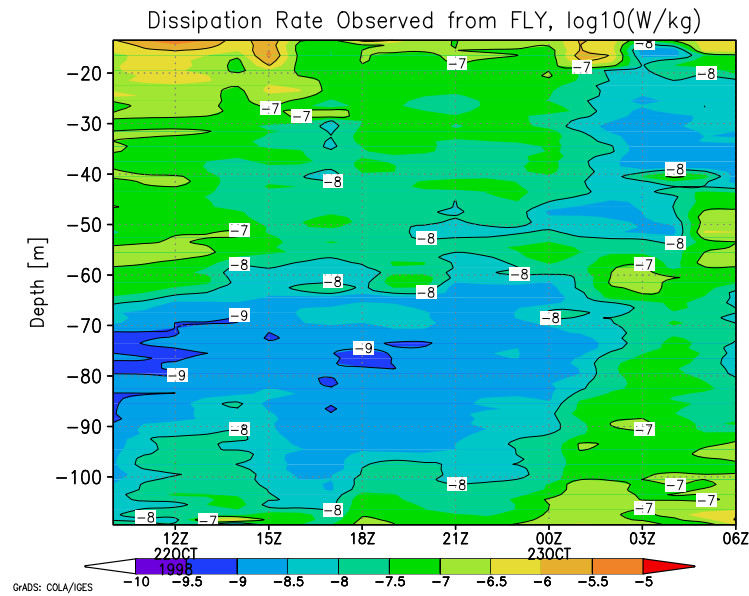
# Modelling tidal flow

## Zero-equation turbulence model:

Phases for M4 tide vs depth

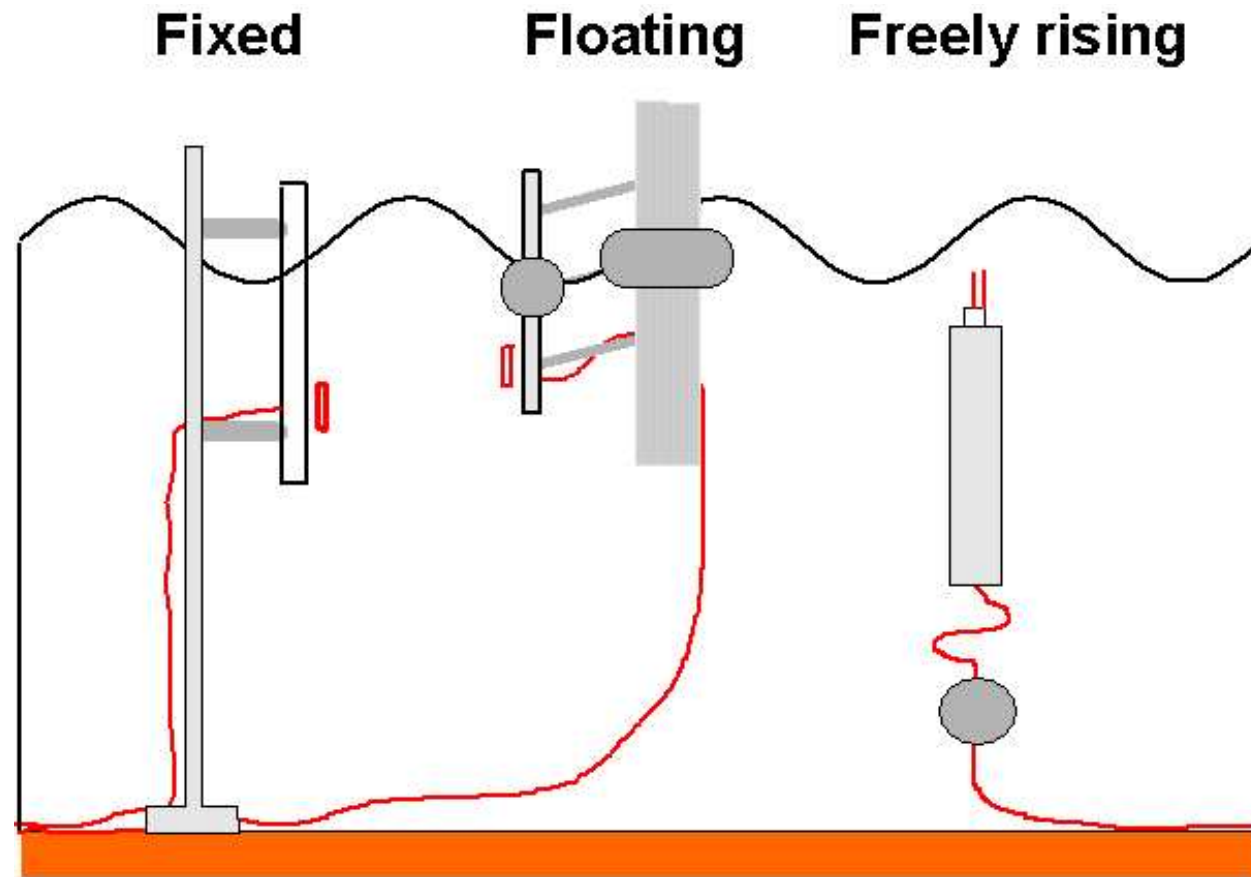


# How big was $\varepsilon$ really ?



From Burchard et al. [2002]

# Turbulence under waves



What does ensemble average mean here ?

# Conclusions ?

To be drawn after the first  
Warnemünde Turbulence Days.