Assignment 3

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1 Problem 1

In this problem we will take a look at how an infinite ocean at rest reacts to a homogeneous wind starting at $t = 0$. The wind acts as a volume force onto the surface mixed layer:

$$X(t, z) = X_0\, \theta(z + H_{mix})\, \theta(t) \tag{1}$$
$$Y(t, z) = 0. \tag{2}$$

Our equations of motion are:

$$\frac{\partial u}{\partial t} - fv = X - ru \tag{3}$$
$$\frac{\partial v}{\partial t} + fu = Y - rv \tag{4}$$

with $r > 0$ being a damping coefficient. The goal of this exercise is to find $u(t)$ and $v(t)$.

(a) Describe in your own words the meaning of each term in eq. (3) and (4).

(b) Introduce the complex velocity U,

$$U = u + iv \tag{5}$$

to reduce the differential equation system to one equation.

(c) Use a Fourier ansatz for $U(t)$ and $X(t)$ to find an expression for $\tilde{U}(\omega)$. We define Fourier transformation by:

$$f(t) = \int_{-\infty}^{\infty} e^{-i\omega t} \tilde{f}(\omega)\, \frac{d\omega}{2\pi}, \qquad \text{and} \qquad \tilde{f}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} f(t)\, dt \tag{6}$$

Hint: The Fourier transform of $f(t) = \theta(t)$ is $\tilde{f}(\omega) = \frac{i}{\omega + i\epsilon}$.

(d) Transform $\tilde{U}(\omega)$ back to $U(t)$ using the definition in eq. (6). This is a complex path integral. Use the Residue theorem to solve it. Is the path integral closed in the upper or lower halfplane? Hint: If you didn't find $\tilde{U}(\omega)$:

$$\tilde{U}(\omega) = \frac{-X_0 \theta(z + H_{mix})}{(\omega + i\epsilon)(\omega - f + ir)} \tag{7}$$

(e) Compute the real and imaginary part of $U(t)$ to find $u(t)$ and $v(t)$. Describe the resulting motion of the velocity field of u and v.