Assignment 7

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1 Problem 1

In quasi-geostrophic theory we introduced the *quasi-geostrohpic stream function* Ψ:

$$\Psi = \frac{p}{f_0}, \quad \text{with} \quad u^{(0)} = -\Psi_y, \quad v^{(0)} = \Psi_x. \tag{1}$$

We found a single equation for Ψ:

$$\frac{d^{(0)}}{dt}\left(\Delta_h \Psi + \beta y + \frac{\partial}{\partial z}\frac{f_0^2}{N^2}\frac{\partial \Psi}{\partial z}\right) = curl \mathcal{F} \tag{2}$$

(a) Show that ignoring external forces and doing a linearization of (2) yields to:

$$\frac{\partial}{\partial t}\left(\Delta_h \Psi + \frac{\partial}{\partial z}\frac{f_0^2}{N^2}\frac{\partial \Psi}{\partial z}\right) + \beta \frac{\partial \Psi}{\partial x} = 0. \tag{3}$$

(b) Do the mode decomposition for Ψ, $\Psi = \tilde{\Psi}(x, y, t)F(z)$, to find:

$$\frac{\partial}{\partial t}\left(\Delta_h \tilde{\Psi} - \lambda^2 f_0^2 \tilde{\Psi}\right) + \beta \frac{\partial \tilde{\Psi}}{\partial x} = 0. \tag{4}$$

Hint: Use the Eigenvalue equation, eq. (2.92) in the lecture, to introduce λ^2 and to get rid of $F(z)$.

(c) Do a Fourier ansatz for $\tilde{\Psi}$:

$$\tilde{\Psi} = \tilde{\Psi_0} e^{i(k_1 x + i k_2 y - \omega t)} \tag{5}$$

to find the dispersion relation of Rossby waves:

$$\omega = -\frac{\beta k_1}{k_1^2 + k_2^2 + R^{-2}} \tag{6}$$

Hint: Use the Rossby radius R, $R = \frac{1}{\lambda f_0}$.

(d) Compute the phase velocity $c_p = \frac{\omega}{k}$ and the group velocity $c_g = \frac{\partial \omega}{\partial k}$ for the zonal (x-direction) component and discuss the results regarding the propagation of phase and energy for long waves, $kR \ll 1$, and short waves, $kR \gg 1$.