Recent advances in oceanic mixed-layer modelling

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Program of presentation

- NSE $\longrightarrow$ Two-equation closure models
- Two-equation closure models
- Langmuir Circulation parameterisations
- General Ocean Turbulence Model (GOTM)
- Examples (observations versus simulations)
- General Estuarine Transport Model (GETM)
- Three-dimensional model study
- Conclusions

Basic approaches

Two approaches to averaged turbulence modelling:

- **Statistical turbulence modelling:** Convert NSE to Friedmann-Keller series, cut-off where suitable and parameterise unknown terms.
- **Empirical turbulence modelling:** Close equations on lowest order and parameterise relevant processes.

NSE $\longrightarrow$ Reynolds equation

Momentum Equation:

$$\partial_t v_i + v_j \partial_j v_i - \nu \partial_{jj} v_i + 2\varepsilon_{ijl}\Omega_j v_l = -\frac{\partial_i p}{\rho_0} - \frac{g_i}{\rho_0}\rho.$$

Reynolds averaging:

$$v_i = \bar{v}_i + \tilde{v}_i, \quad \rho = \bar{\rho} + \tilde{\rho}, \ldots$$

Reynolds Equation:

$$\partial_t \bar{v}_i + \bar{v}_j \partial_j \bar{v}_i - \partial_j \left(\nu \partial_j \bar{v}_i - \langle \tilde{v}_j \tilde{v}_i \rangle\right) + 2\varepsilon_{ijl}\Omega_j \bar{v}_l = -\frac{\partial_i \bar{p}}{\rho_0} - g_i \frac{\bar{\rho}}{\rho_0}.$$

Reynolds Stress Equation

$$
\begin{aligned}
\partial_t \langle \tilde{v}_i \tilde{v}_j \rangle + \partial_l \left(\bar{v}_l \langle \tilde{v}_i \tilde{v}_j \rangle + \langle \tilde{v}_l \tilde{v}_i \tilde{v}_j \rangle - \nu \partial_l \langle \tilde{v}_i \tilde{v}_j \rangle \right) \\
= \underbrace{-\partial_l \bar{v}_i \langle \tilde{v}_l \tilde{v}_j \rangle - \partial_l \bar{v}_j \langle \tilde{v}_l \tilde{v}_i \rangle}_{P_{ij}} \\
- \underbrace{2\Omega_l (\varepsilon_{ilm} \langle \tilde{v}_j \tilde{v}_m \rangle + \varepsilon_{jlm} \langle \tilde{v}_i \tilde{v}_m \rangle)}_{\Omega_{ij}} \\
- \underbrace{\frac{1}{\rho_0} \{ g_i \langle \tilde{v}_j \tilde{\rho} \rangle + g_j \langle \tilde{v}_i \tilde{\rho} \rangle \}}_{B_{ij}} - \underbrace{\frac{1}{\rho_0} (\langle \tilde{v}_i \partial_j \tilde{p} + \tilde{v}_j \partial_i \tilde{p} \rangle)}_{\Pi_{ij}} \\
- \underbrace{2\nu \langle (\partial_l \tilde{v}_j)(\partial_l \tilde{v}_i) \rangle}_{\varepsilon_{ij}} .
\end{aligned}
$$

Algebraic SMCs

The following steps lead to different types of second-moment closures:

- Empirical closures of pressure-strain correlators.
- Neglect or simplifi cation of advective and diffusive fluxes of second-moments.
- Neglect of rotational terms in the second-moment equations.
- Boundary layer assumption (neglect of horizontal gradients and non-hydrostatic effects).
- ... and many more details ...

Algebraic SMCs

Turbulent Fluxes:

$$\langle \tilde{u}\tilde{w} \rangle = -\nu_t \partial_z \bar{u} , \quad \langle \tilde{w}\tilde{T} \rangle = -\nu_t' \partial_z \bar{T}$$

Eddy Viscosity / Eddy Diffusivity:

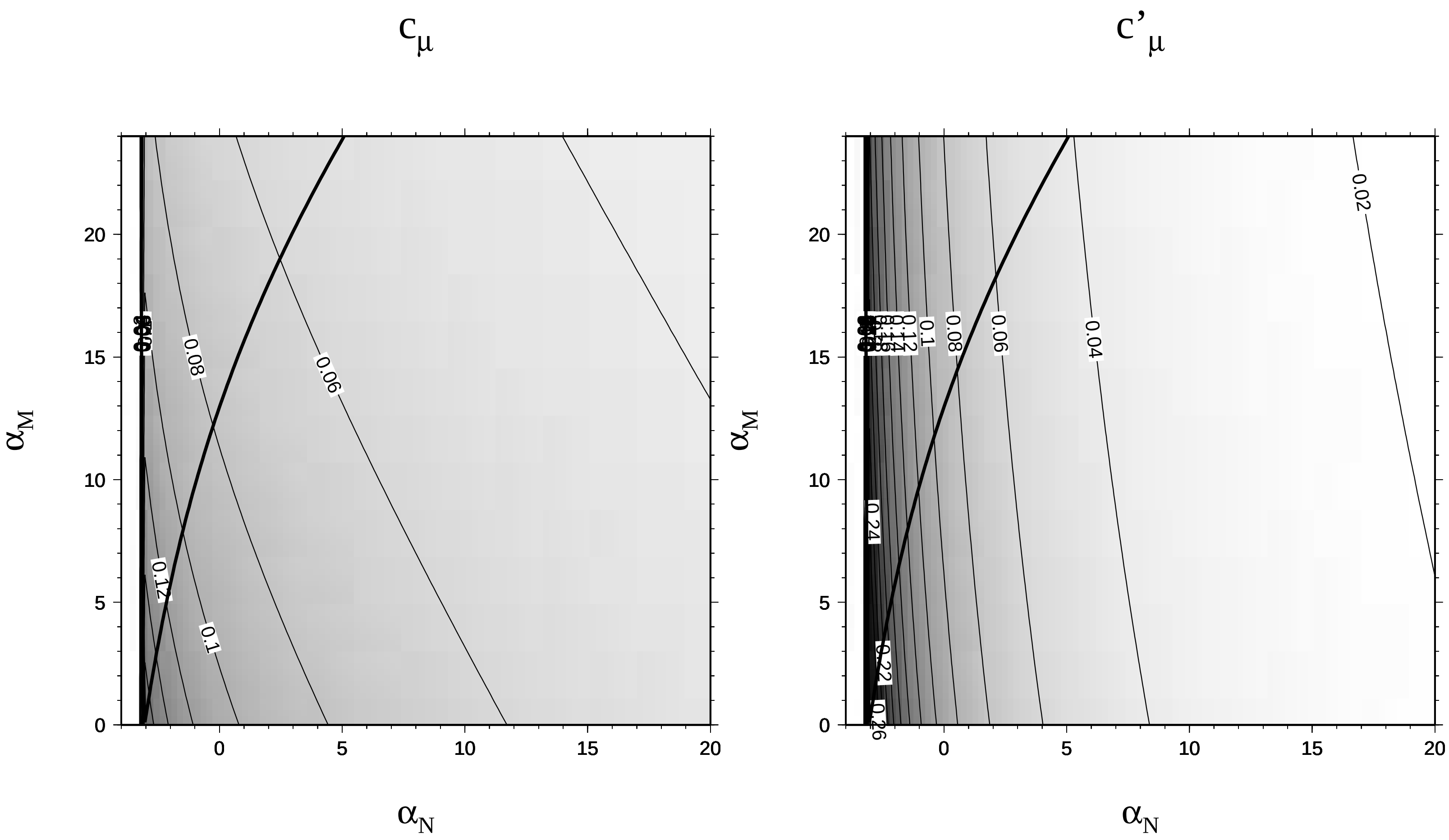
$$\nu_t = c_\mu(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon} , \quad \nu_t' = c_\mu'(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon} .$$

Shear Number, Buoyancy Number:

$$\alpha_M = \frac{k^2}{\varepsilon^2} M^2 , \quad \alpha_N = \frac{k^2}{\varepsilon^2} N^2 .$$

Stability Functions

Canuto et al. [2001]:



Exact TKE-Equation

$$\partial_t k + \partial_j \left(\bar{v}_j k + \langle \tilde{v}_j \tfrac{1}{2} \tilde{v}_i^2 \rangle - \nu \partial_j k + \frac{1}{\rho_0} \langle \tilde{v}_j \tilde{p} \rangle \right)$$

$$= \underbrace{- \langle \tilde{v}_j \tilde{v}_i \rangle \partial_i \bar{v}_j}_{P} \underbrace{- \frac{g}{\rho_0} \langle \tilde{v}_3 \tilde{\rho} \rangle}_{B} - \underbrace{\nu \langle (\partial_j \tilde{v}_i)^2 \rangle}_{\varepsilon},$$

This TKE equation will be modelled as it is given above, the only parameterisations needed are for the turbulent flux terms, for which usually the down-gradient approximation is used.

Dissipation equation

Exact form (e.g. *Wilcox* [1998]):

$$
\begin{aligned}
&\partial_t \varepsilon + \partial_j \left(\bar{v}_j \varepsilon + \langle \tilde{v}_j \nu (\partial_j \tilde{v}_i)^2 \rangle - \nu \partial_j \varepsilon + 2\frac{\nu}{\rho_0} \langle \partial_i \tilde{v}_j \partial_i \tilde{p} \rangle \right) \\
&\qquad = \underbrace{-2\nu \partial_j \bar{v}_i \left(\langle \partial_i \tilde{v}_k \partial_j \tilde{v}_k \rangle + \langle \partial_k \tilde{v}_i \partial_k \tilde{v}_j \rangle \right) - 2\nu \partial_{jk} \bar{v}_i \langle \tilde{v}_k \partial_j \tilde{v}_i \rangle}_{P_\varepsilon} \\
&\qquad \underbrace{-2\nu \frac{g}{\rho_0} \partial_j \langle \tilde{v}_3 \partial_j \tilde{\rho} \rangle}_{B_\varepsilon} - \underbrace{2\nu \left(\langle \partial_j \tilde{v}_i \partial_k \tilde{v}_i \partial_j \tilde{v}_k \rangle + \nu \langle (\partial_{ij} \tilde{v}_k)^2 \rangle \right)}_{\varepsilon_\varepsilon}
\end{aligned}
\qquad (1)
$$

k-ε model (*Launder and Spalding* [1972]):

$$
\partial_t \varepsilon - \partial_z \left(\frac{\nu_t}{\sigma_\varepsilon} \partial_z \varepsilon \right) = \frac{\varepsilon}{k} \left(c_{\varepsilon 1} P + c_{\varepsilon 3} B - c_{\varepsilon 2} \varepsilon \right).
$$

Mellor-Yamada model

General relation between k, ε and L:

$$L = c_\mu^{3/4} \frac{k^{3/2}}{\varepsilon}$$

k-kL model (*Mellor and Yamada* [1982]):

$$\partial_t (kL) - \partial_z (S_l \partial_z (kL)) =$$

$$\frac{L}{2} \left[E_1 P + E_3 B - \left(1 + E_2 \left(\frac{L}{L_z} \right)^2 \right) \varepsilon \right] .$$

Length scale equations (cont'd)

Other approaches are using equations for $\omega = \varepsilon/k$ (k-ω model), k/ε, k^2/ε, ..., so why not using the generalised approach of a $k^n\varepsilon^m$ equation ?

Generic length scale equation
(*Umlauf and Burchard* [2003]):

$$\partial_t \left(k^n\varepsilon^m\right) - \partial_z \left(\frac{\nu_t}{\sigma_{nm}} k^n\varepsilon^m\right) =$$

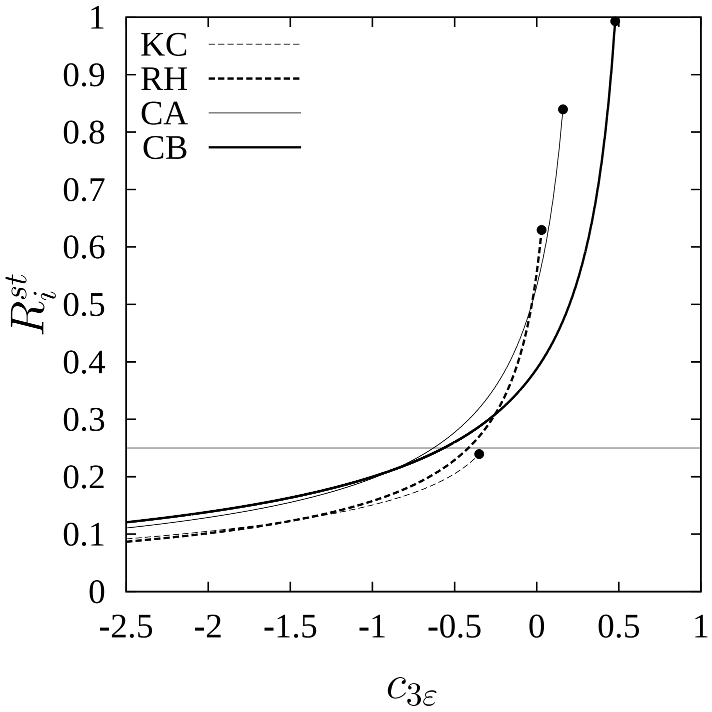
$$k^{n-1}\varepsilon^m \left(c_{nm1}P + c_{nm3}B - c_{nm2}\varepsilon\right)$$

This works without correction term only for $m \geq 0$.

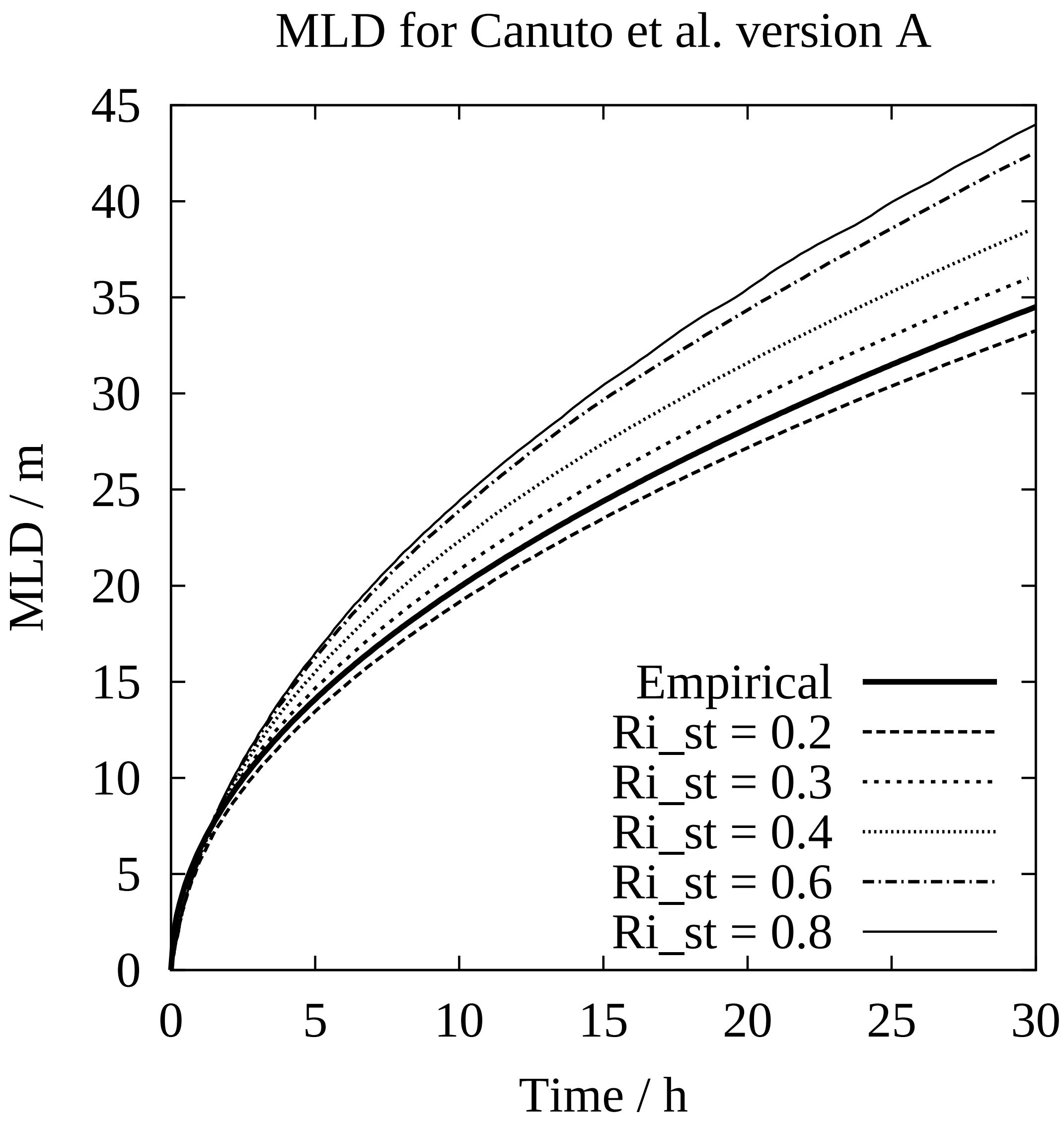
Total equilibrium (k-ε)

$$\dot{k} = \dot{\varepsilon} \Longrightarrow R_i = R_i^{st} = \frac{c_\mu}{c'_\mu} \cdot \frac{c_{2\varepsilon} - c_{1\varepsilon}}{c_{2\varepsilon} - c_{3\varepsilon}}.$$

$R_i^{st} \approx 0.25$: Steady-state Richardson number.



Kato-Phillips experiment



Langmuir Circulation

Stokes drift velocity:

$$\{U_s, V_s\} = \left\{ \left(\frac{k_x h_s}{2}\right)^2, \left(\frac{k_y h_s}{2}\right)^2 \right\} \sqrt{\frac{g}{k}} \exp(-2kz) \qquad (1)$$

Modifi ed Reynolds eqations:

$$\partial_t \bar{v}_i + \bar{v}_j \partial_j \bar{v}_i - \partial_j (\nu \partial_j \bar{v}_i - \langle \tilde{v}_j \tilde{v}_i \rangle) + 2\varepsilon_{ijl} \Omega_j \bar{v}_l$$

$$= -\frac{\partial_i \bar{p}}{\rho_0} - g_i \frac{\bar{\rho}}{\rho_0} + \varepsilon_{ijl} \{ (U_s)_j \varepsilon_{lmn} \partial_m \bar{u}_n \}.$$

Vortex force term:

$$\varepsilon_{ijl} \{ (U_s)_j \varepsilon_{lmn} \partial_m \bar{u}_n \} = \vec{U}_s \times (\vec{\nabla} \times \vec{\bar{u}}). \qquad (2)$$

Langmuir Circulation, cont'd

Langmuir Circulation (LC) in TKE models

TKE-equation (*Kantha and Clayson*, [2003]):

$$\partial_t k - \partial_z \left(\frac{\nu_t}{\sigma_k} \partial_z k \right)$$

$$= \underbrace{\nu_t \left((\partial_z \bar{u})^2 + (\partial_z \bar{v})^2 \right)}_{P} \underbrace{+2\nu_t \left(U_s \partial_z \bar{u} + V_s \partial_z \bar{v} \right)}_{P_L}$$

$$\underbrace{+\frac{g}{\rho_0} \nu_t' \partial_z \bar{\rho}}_{B} - \varepsilon .$$

Empirical model: KPP

K-Profile Parameterisation Model

$$\langle \tilde{w}\tilde{x} \rangle = \underbrace{-K_x \partial_z \bar{x}}_{\text{local}} + \underbrace{K_x \gamma_x}_{\text{non-local}}, \tag{3}$$

$$K_x = h(t) w_x(z,t) G_x(z,t) \tag{4}$$

$h(t)$	mixed-layer depth	
$w_x(z,t) = \frac{\kappa u_*(t)}{\phi_x(z,t)}$	turbulent velocity scale	(5)
$G_x(z,t)$	shape function	
$\partial_z \bar{x} = -\frac{\langle \tilde{w}\tilde{x} \rangle_0}{u_*} \frac{\phi_x(z/L)}{\kappa \lvert z \rvert}$	structure function ϕ_x	

KPP continued

Langmuir Circulation (LC) in KPP

Enhanced mixing due to LC by *Smyth et al.* [2002]:

$$w_x(z,t) = \frac{\kappa u_*(t)}{\phi_x(z,t)} \left\{ 1 + \frac{C_w(u_*, w_*)}{La^4} \right\}^{1/2} \tag{6}$$

with

$$C_w(u_*, w_*) = C_{w0} \left[\frac{u_*^3}{u_*^3 + 0.6 w_*^3} \right]^l , \tag{7}$$

$$\begin{aligned} &\text{Deardorff velocity scale:} \quad && w_* = (\kappa B_0 h)^{1/3}, \\ &\text{Langmuir number:} && La = \left(\frac{u_*}{|\vec{U}_s(0)|} \right)^{1/2} \end{aligned} \tag{8}$$

GOTM, http://www.gotm.net

GO TM

Challenge
- Aim
- The Idea
- Key features

Software
- Fortran code
- Test cases
- Forcing
- How to run?

Information
- What's New
- Publications
- E-mail list
- FAQ
- User Group
- Hot Links
- Who's Who?
- Guestbook

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General Ocean Turbulence Model

GOTM is a one-dimensional numerical model developed and supported by a core team of ocean modellers. GOTM aims at simulating accurately vertical exchange processes in the marine environment where mixing is known to play a key role. GOTM is freely available under the GPL (Gnu Public License).

The interested user can download the source code, a set of test cases (Papa, November, Flex, ...) and a comprehensive report.

You are warmly invited to join the GOTM mailing list and send any comments/questions to the GOTM team or become a GOTM contributor. The GOTM developers are grateful to their sponsors.

Page "www.gotm.net" maintained by webmaster. Last update: 10/28/00 18:10:02

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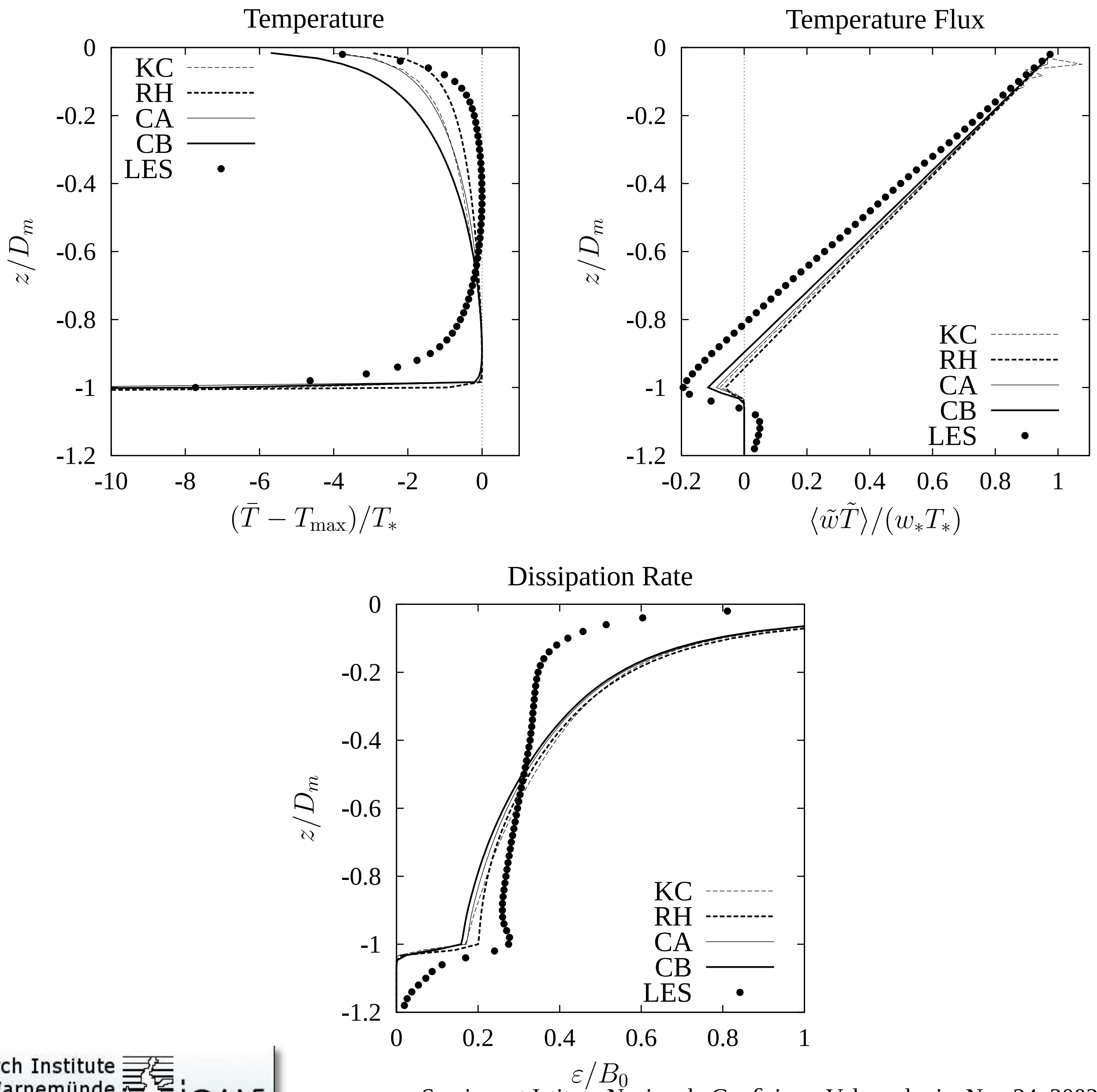
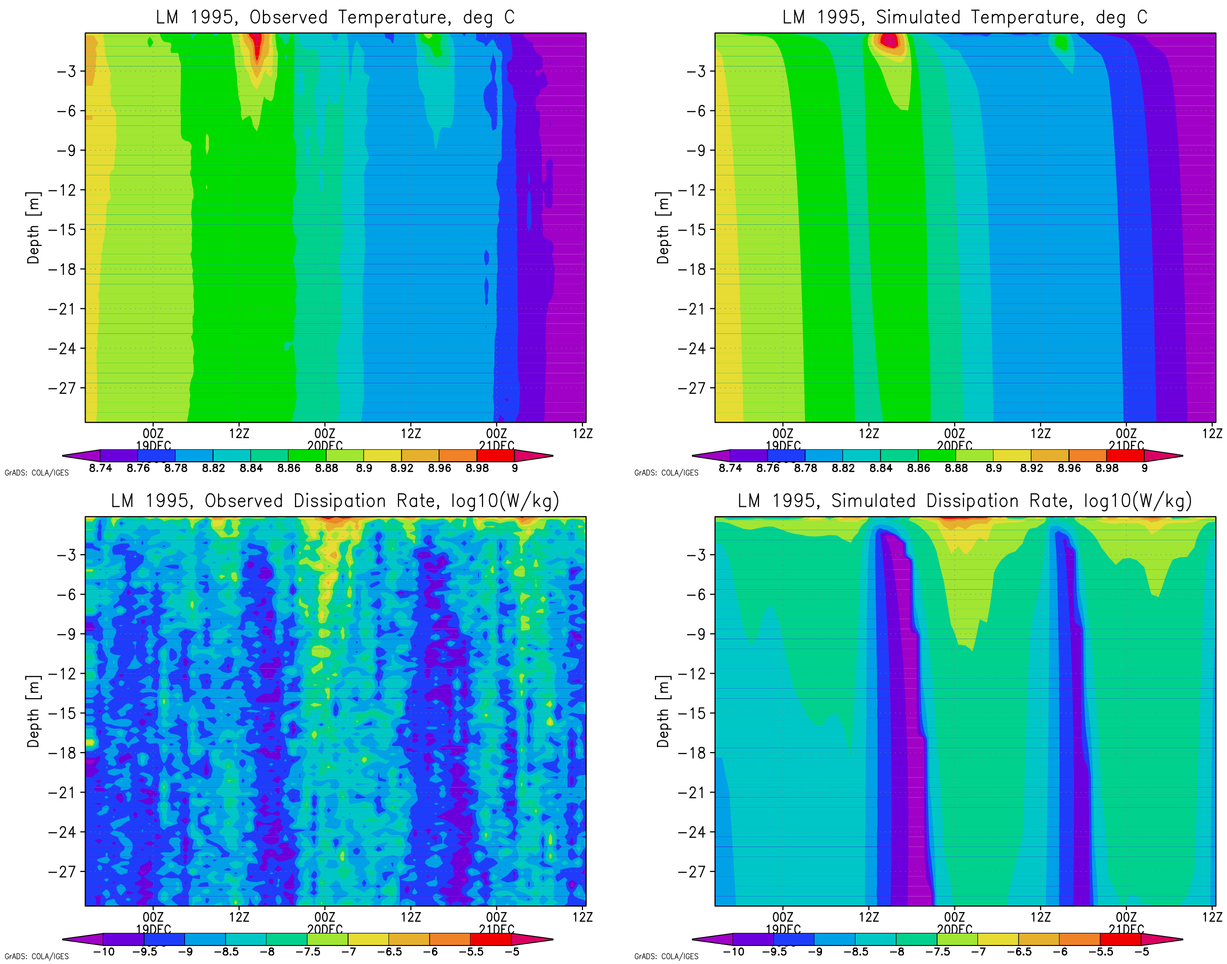
Free Convection



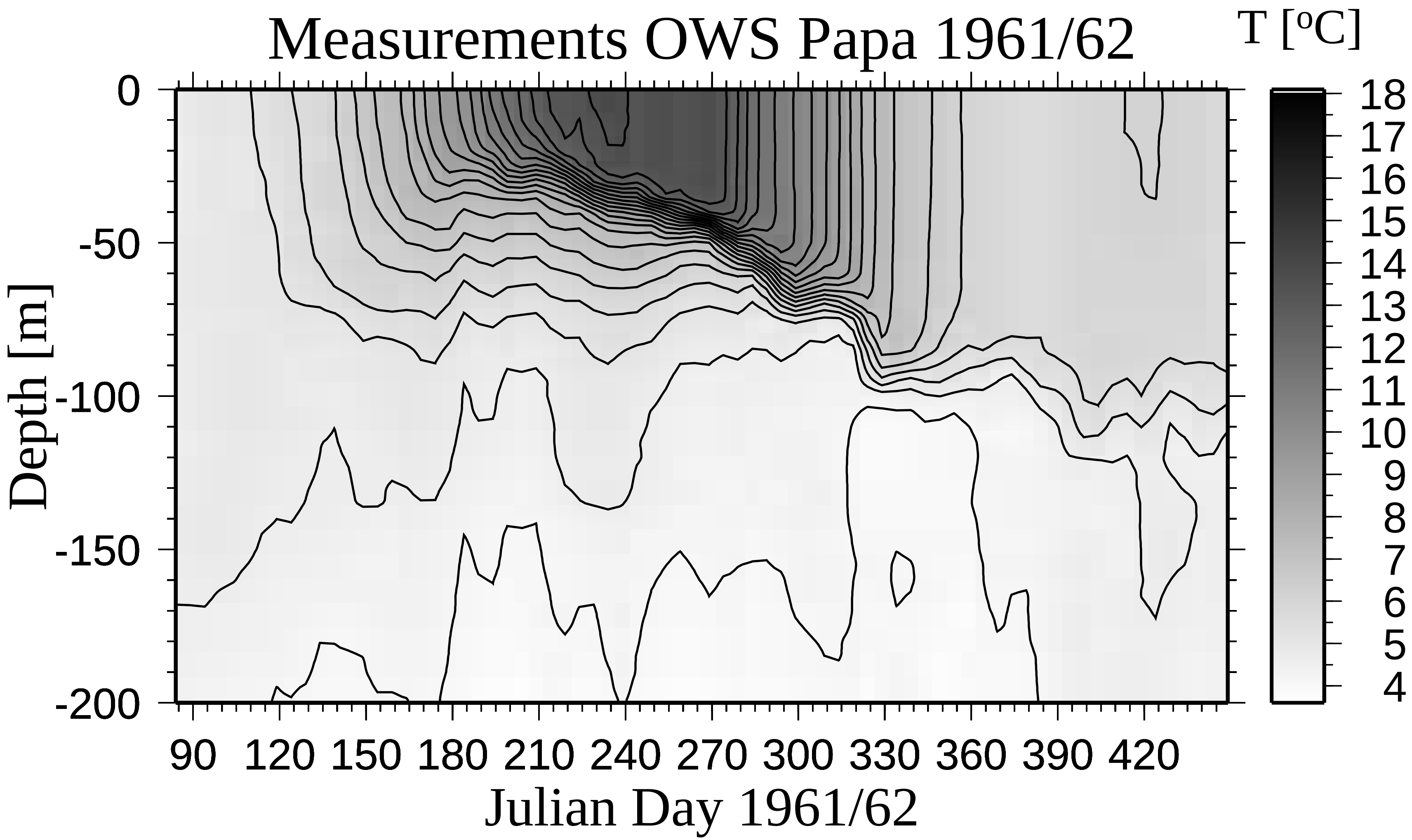
Lago Maggiore, Italy

Observations and simulations of T and ε (Stips et al. [2002])



Open Ocean: OWS Papa

Temperature Observations



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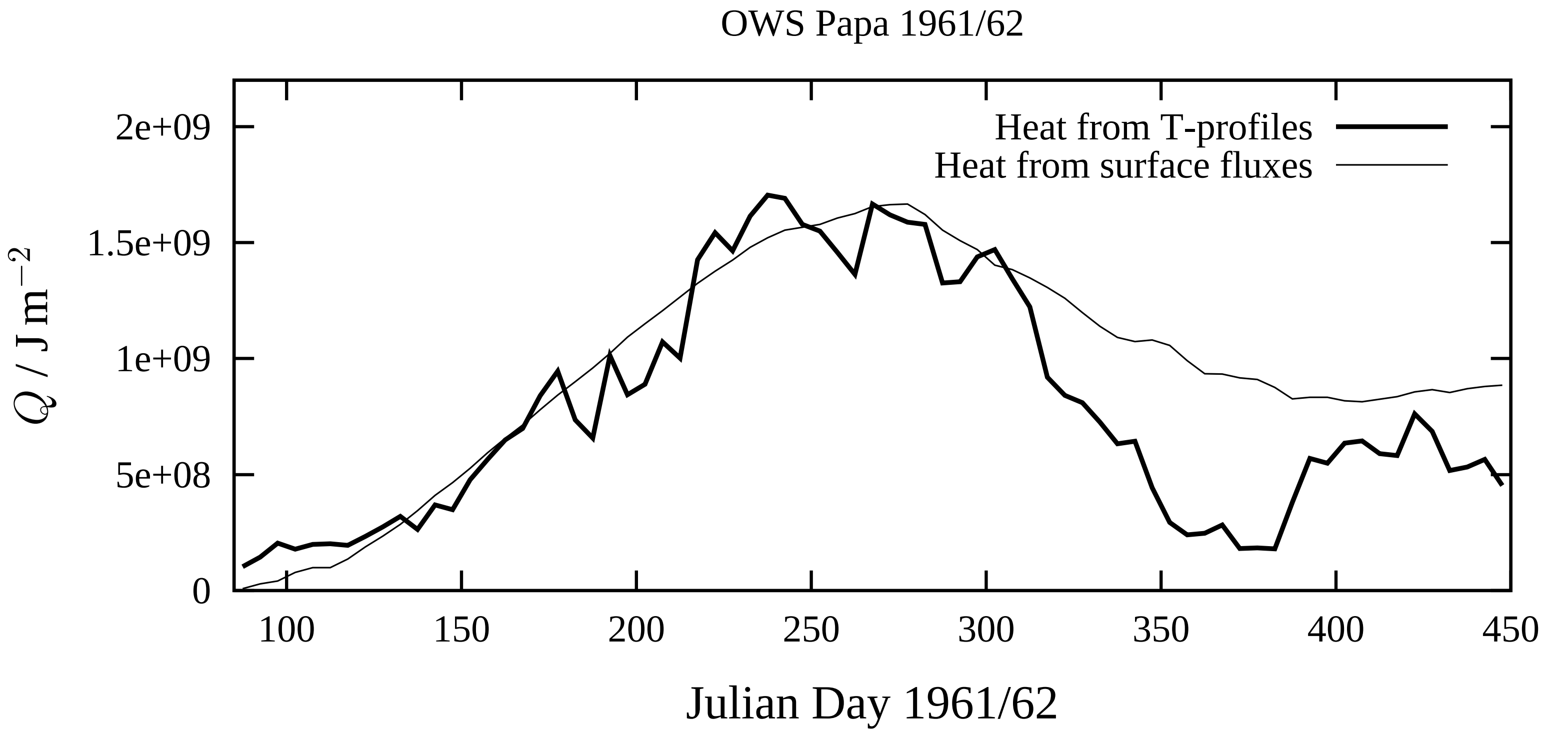
Heat budget



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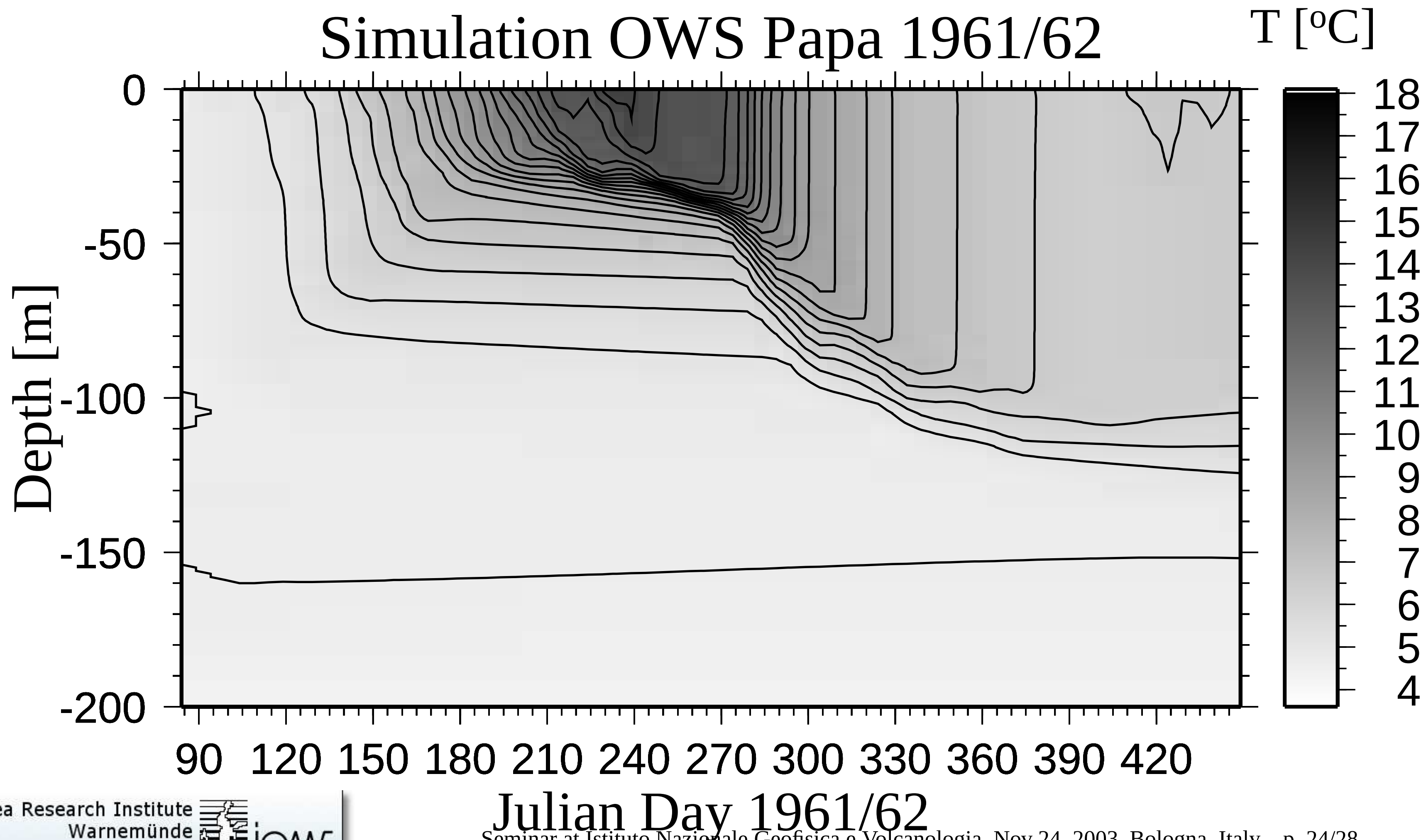
Temperature Simulations: k-ε model with alg. SMC



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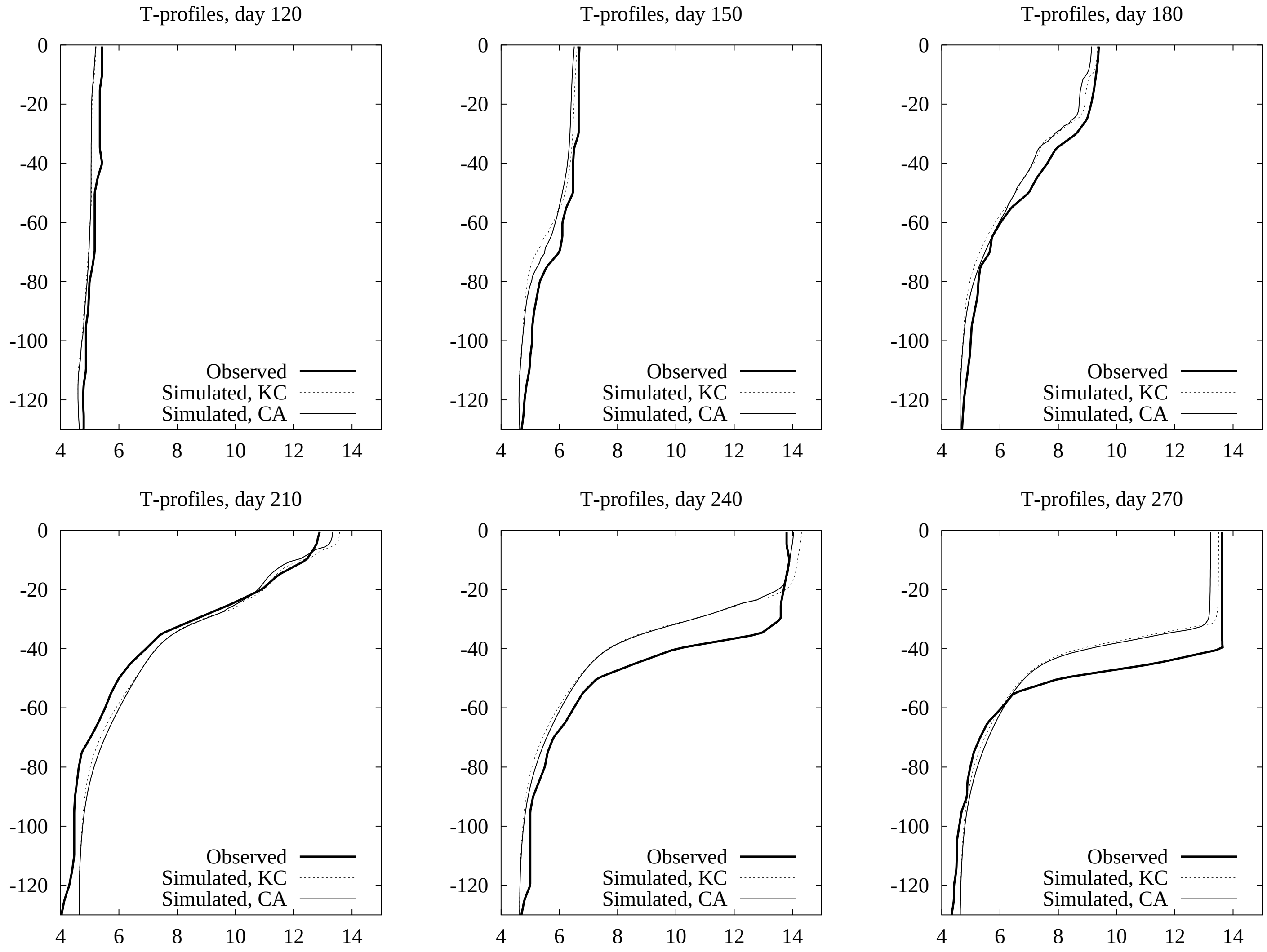
Temperature profiles



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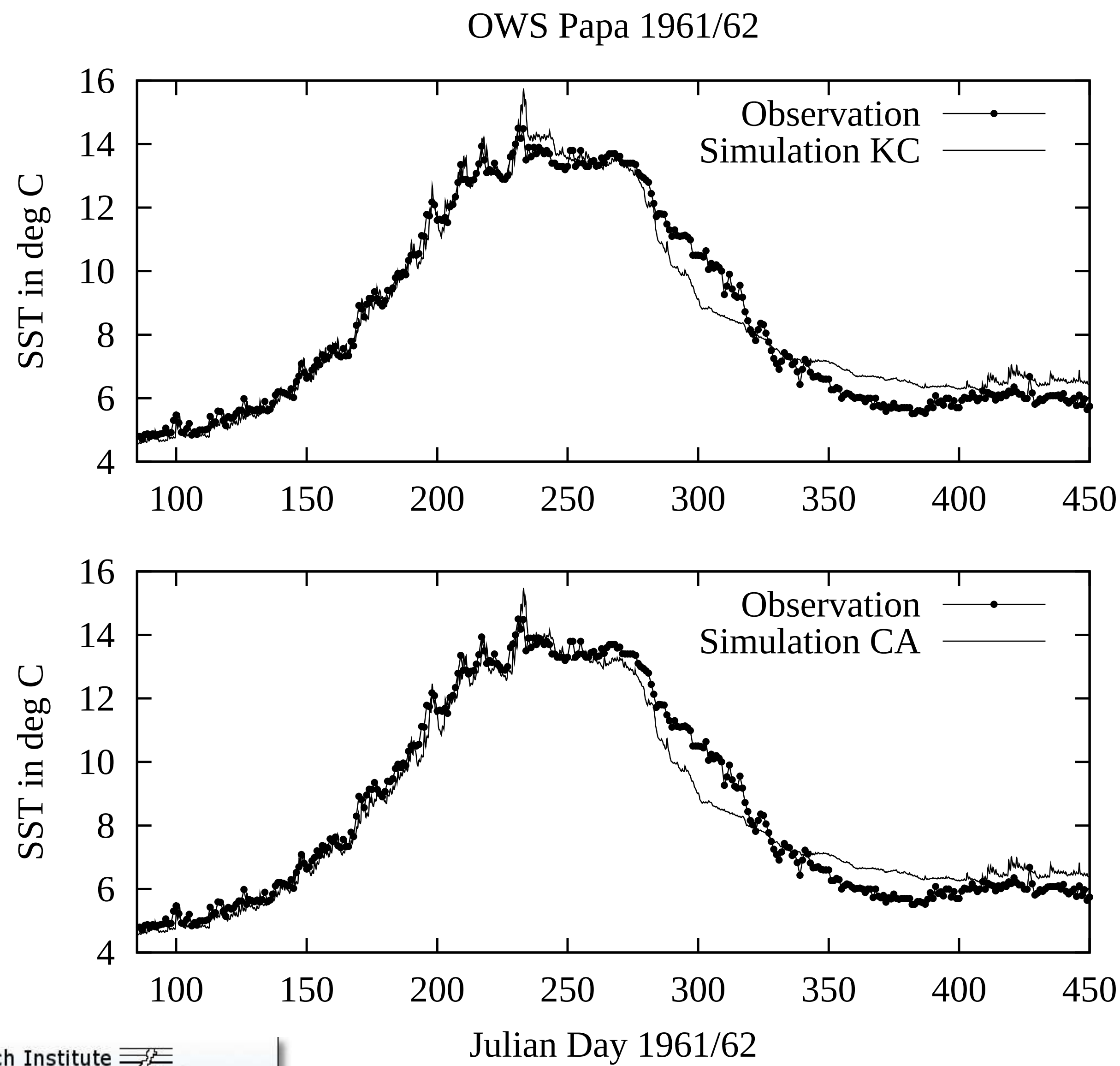
Sea Surface Temperature



GETM

General Estuarine Transport Model

(Burchard & Bolding [2002])

- General vertical coordinates
- Horizontal curvilinear coordinates
- Uses GOTM as turbulence model
- Drying & flooding
- Using MPI for parallel runs
- Public Domain

Conclusions

- Reynolds decomposition provides a physically sound framework for developing turbulence models.
- Empirical parameters allow for calibrating such models to the real world.
- Langmuir Circulation parameterisations in KPP and TKE models are not mature yet.
- For mixed layers, two-equation models with algebraic SMCs are sufficient for reproducing observations.
- These models are economic enough for using them in 3D models for oceanic applications.