Observations and numerical simulations of small-scale turbulence in oceanic boundary layers

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Program of presentation

- Some basic statistical moments of turbulence to be observed and simulated
- In-situ observations of marine turbulence
- Principles of numerical turbulence modelling
- Some example observations and simulations
- Surface wave breaking
- Conclusions
Temperature profiles in Lago Maggiore

\[ T \quad \langle T \rangle \quad \tilde{T} \]

\[ T / ^\circ C \quad T / ^\circ C \quad T / ^\circ C \]

pers. comm. Adolf Stips
Some statistical moments

Turbulent kinetic energy (TKE): 
\[ k = \frac{1}{2} \sum_{i=1}^{3} \langle \tilde{u}_i^2 \rangle \]

Dissipation rate of TKE: 
\[ \varepsilon = \nu \sum_{i,j=1}^{3} \left\langle \left( \frac{\partial \tilde{u}_i}{\partial x_j} \right)^2 \right\rangle \]

Shear production rate of TKE: 
\[ P = - \sum_{i,j=1}^{3} \langle \tilde{u}_j \tilde{u}_i \rangle \partial_i \langle u_j \rangle \]
Micro-structure shear probe

Sensor shaft Ø 11 mm
Holes
Piezoceramic beam
Waterproof isolation
Metallic cap
Cantilever

Cross force
Airfoil Ø 3 mm
length 3.5 mm

Profiling velocity U

pers. comm. Hartmut Prandke
Power spectrum from shear probe

pers. comm. Adolf Stips
High-resolution ADCP

\[ u_3 = u \sin \theta + w \cos \theta; \quad u_4 = u \sin \theta + w \cos \theta; \quad (1) \]

\[ \langle \ddot{u}_3^2 \rangle = \langle \ddot{u}^2 \rangle \sin^2 \theta + \langle \ddot{w}^2 \rangle \cos^2 \theta + 2 \langle \ddot{u} \ddot{w} \rangle \sin \theta \cos \theta \]
\[ \langle \ddot{u}_4^2 \rangle = \langle \ddot{u}^2 \rangle \sin^2 \theta + \langle \ddot{w}^2 \rangle \cos^2 \theta - 2 \langle \ddot{u} \ddot{w} \rangle \sin \theta \cos \theta \quad (2) \]

\[ \langle \ddot{u} \ddot{w} \rangle = \frac{\langle \ddot{u}_3^2 \rangle - \langle \ddot{u}_4^2 \rangle}{4 \sin \theta \cos \theta} \quad (3) \]
Particle Image Velocimetry (PIV)

pers. comm. Submersible PIV Group (Katz et al.),
http://cloudbase.me.jhu.edu/spiv/
Averaging procedures

Spatial average:

\[ \langle T(\mathbf{x}, t) \rangle = \frac{1}{|V|} \iiint_{V} T(\xi, t) \, d\xi \]

Ensemble average:

\[ \langle T(\mathbf{x}, t) \rangle = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} T_i(\mathbf{x}, t) \]

Application of ensemble average:

\[ \langle \partial_z (uw) \rangle = \partial_z \langle (\bar{u} + \tilde{u})(\bar{w} + \tilde{w}) \rangle = \partial_z (\bar{u}\bar{w}) + \partial_z \langle \tilde{u}\tilde{w} \rangle \]
Algebraic second-moment closures

Turbulent fluxes:

\[ \langle \tilde{u} \tilde{w} \rangle = -\nu_t \partial_z \tilde{u}, \quad \langle \tilde{w} \tilde{T} \rangle = -\nu'_t \partial_z \tilde{T} \]  \hspace{1cm} (1)

Eddy viscosity / eddy diffusivity:

\[ \nu_t = c_\mu(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon}, \quad \nu'_t = c'_\mu(\alpha_M, \alpha_N) \frac{k^2}{\varepsilon}. \]  \hspace{1cm} (2)

Shear and buoyancy number:

\[ \alpha_M = \frac{k^2}{\varepsilon^2} M^2, \quad \alpha_N = \frac{k^2}{\varepsilon^2} N^2. \]  \hspace{1cm} (3)
Two-equation turbulence models

Turbulent kinetic energy (TKE) $k$:

$$ \partial_t k + \partial_z F(k) = P + B - \varepsilon, \quad (4) $$

Dissipation rate of TKE $\varepsilon$:

$$ \partial_t \varepsilon + \partial_z F(\varepsilon) = \frac{\varepsilon}{k} \left( c_{\varepsilon 1} P + c_{\varepsilon 3} B - c_{\varepsilon 2} \varepsilon \right), \quad (5) $$

Taylor scaling for macro length scale $L$:

$$ L = c_{\mu}^{3/4} \frac{k^{3/2}}{\varepsilon} \quad (6) $$
**Two-equation turbulence models (cont’d)**

**Generic length scale equation** (*Umlauf and Burchard [2002]*):

\[
\partial_t (k^n \varepsilon^m) + \partial_z F (k^n \varepsilon^m) =
\]

\[
k^{n-1} \varepsilon^m (c_{nm1} P + c_{nm3} B - c_{nm2} \varepsilon)
\]

(7)

**Other length scale related transport equations for**

- \( kL \) (*Mellor & Yamada [1974, 1982]*)
- \( \omega = \varepsilon/k \) (*Wilcox [1988, 1998]*)
GOTM is a one-dimensional numerical model developed and supported by a core team of ocean modellers. GOTM aims at simulating accurately vertical exchange processes in the marine environment where mixing is known to play a key role. GOTM is freely available under the GPL (Gnu Public License).

The interested user can download the source code, a set of test cases (Papa, November, Flex, ...) and a comprehensive report.

You are warmly invited to join the GOTM mailing list and send any comments/questions to the GOTM team or become a GOTM contributor. The GOTM developers are grateful to their sponsors.
Northern North Sea

Bathymetry and locations of stations
Northern North Sea

Wind and tidal forcing

Surface stress at station NNS

Bed stress at station NNS

Date in 1998

ADCP

Model
Liverpool Bay

Temperature and salinity transsects

Fig 2.a  Temperature (Degrees C)

Fig 2.b  Salinity (PSU)

Rippeth, Fisher, Simpson [2001]
Liverpool Bay

Observed and simulated dissipation rate

Simpson, Burchard, Fisher, Rippeth [2002]
Lake Lago Maggiore

Surface heat fluxes and temperature

Observed sea surface temperature

Calculated surface heat flux

Date in 1995

SST / °C

Date in 1995

$Q / \text{W m}^{-2}$
Lake Lago Maggiore

Observations and simulations of $T$ and $\varepsilon$ (Stips et al. [2002])
Modelling the wave-enhanced layer

$\frac{(z - z_0)}{H_s}$

$\frac{\varepsilon}{(c_w u^3)} / H_s$

Terray et al. [1996]
Drennan et al. [1996]
Anis and Moun [1995]
$k-\varepsilon$ model, $z_0/H_s = 0.5$
$k-\omega$ model, $z_0/H_s = 0.5$
Log-Law

Umlauf, Burchard, Hutter [2002]
Conclusions

- Observing and numerically simulating statistical properties of marine turbulence is based on similar principles.
- In recent years, observations and model results are converging to each other.
- Consideration of non-standard effects such as surface and internal wave breaking and Langmuir circulation is possible, but still remains a future challenge.