Lagrangian Particle Modelling in Inhomogeneous Turbulent Flow

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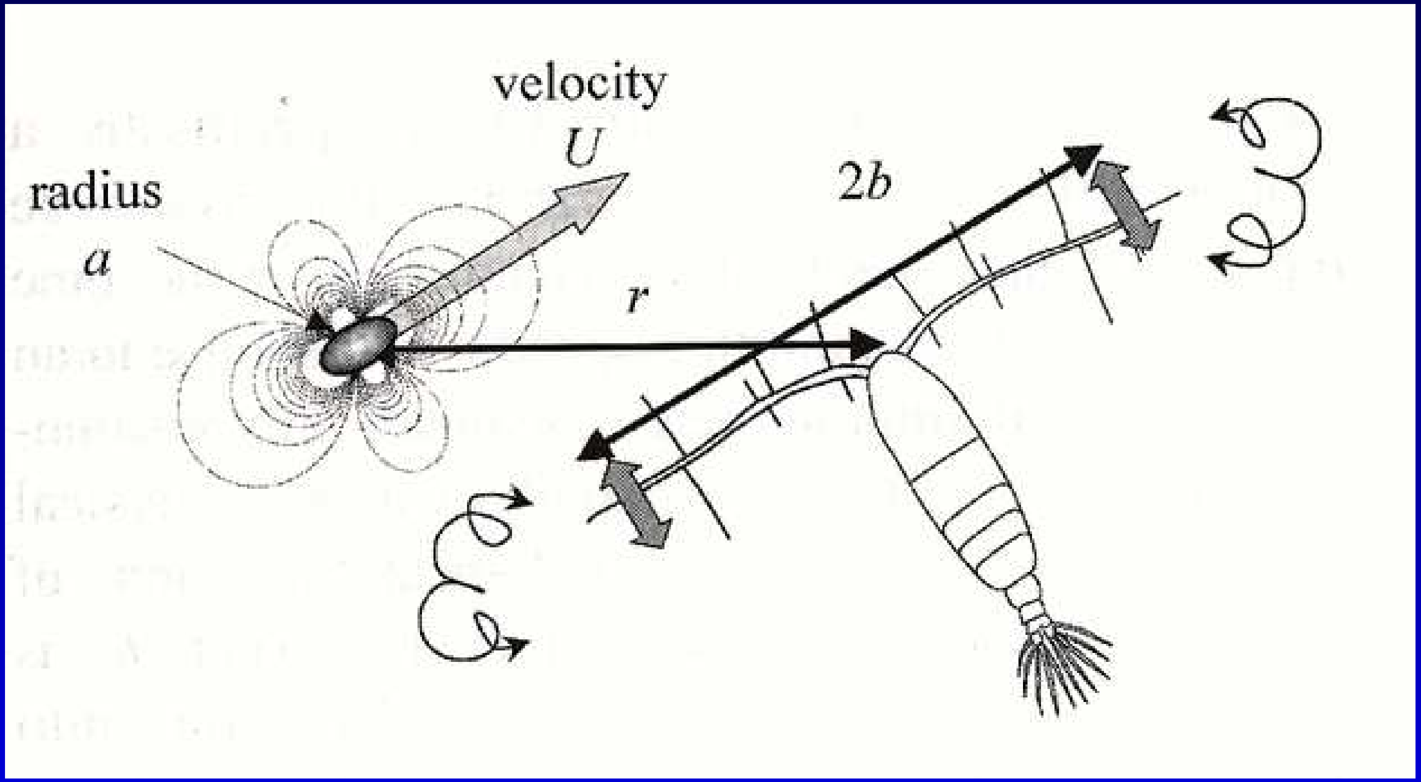
Contents

- Random walk consistent with diffusion
- Basic test: homogeneous concentration
- Less basic test: Rouse profile
- Advanced test: mussel filtration
- Project sketch: AlgaLag
- What's next ?

Motivation

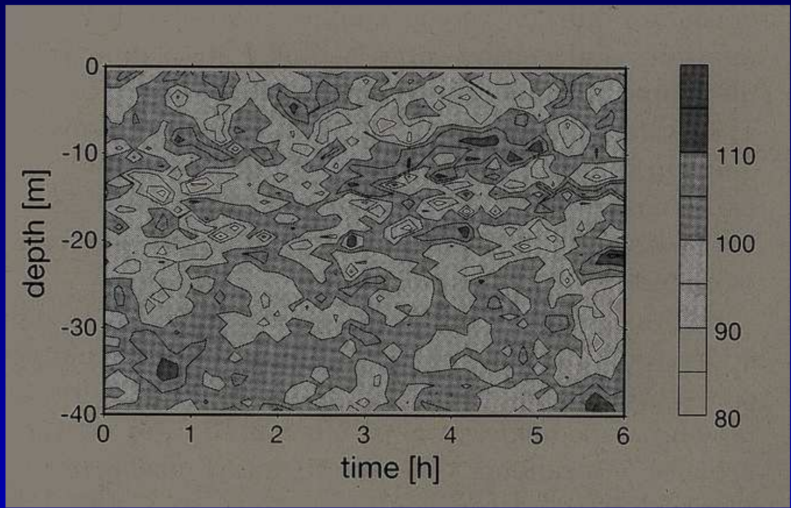
Complex *particle-particle* interaction:

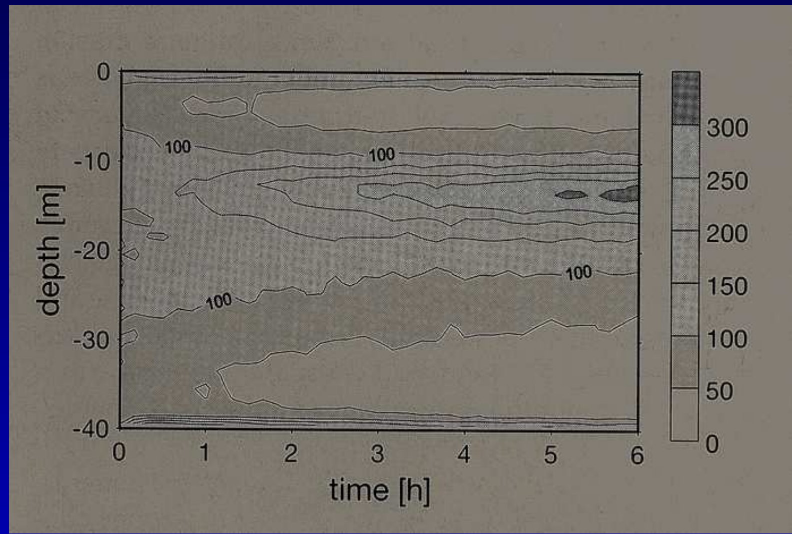


Any chance to model this for a water column ?

Motivation

Consistent (left) and inconsistent (right) random walk:





(Based on viscosity profi le with surface and bottom mixed layer and minimum at $z = -10$ m)

Reference: Visser [1997]

Random walk

Random walk consistent with diffusion equation:

$$\partial_t C - \partial_z \left(\nu_t \partial_z C \right) = 0. \tag{1}$$

$$\begin{aligned} z_i^{n+1} &= z_i^n + \partial_z \nu_t (z_i^n) \Delta t \\ &+ R \left\{ 2 r^{-1} \nu_t (z_i^n + \tfrac{1}{2} \partial_z \nu_t (z_i^n) \Delta t) \Delta t \right\}^{1/2} \end{aligned} \tag{2}$$

R: random process with $\langle R \rangle = 0$ and $\langle R^2 \rangle = r$.

z_i^n : vertical position of particle i at time step n.

Reference: Visser [1997]

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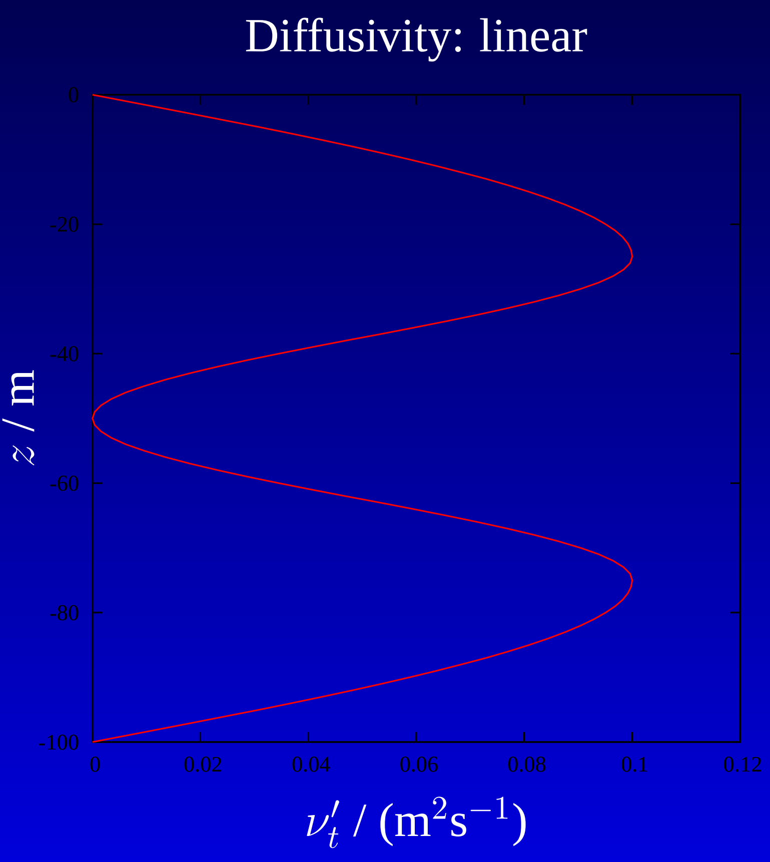
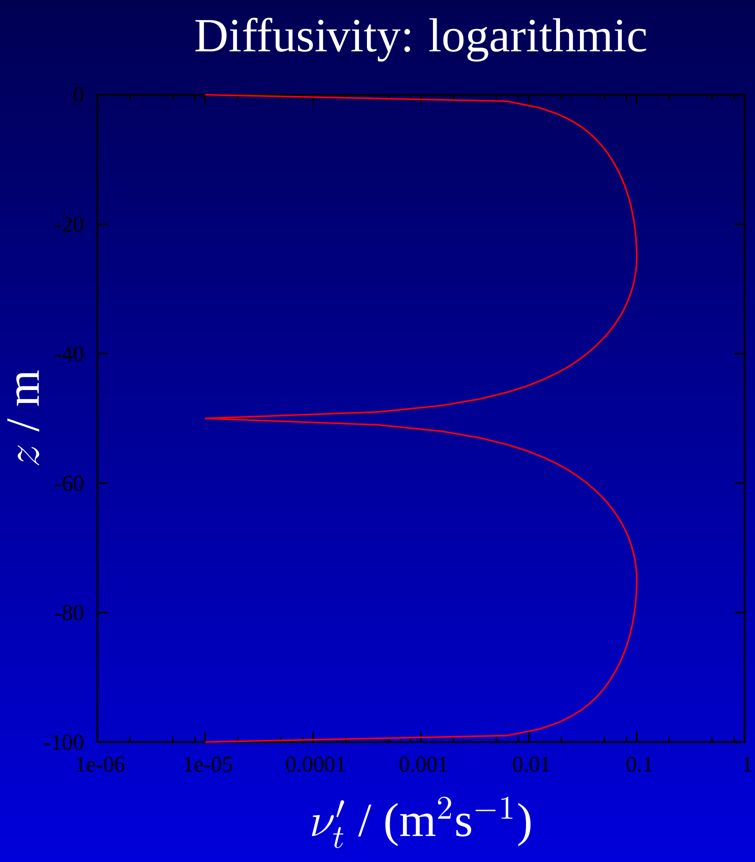
Homogeneous concentration

Prescribed diffusivity profile

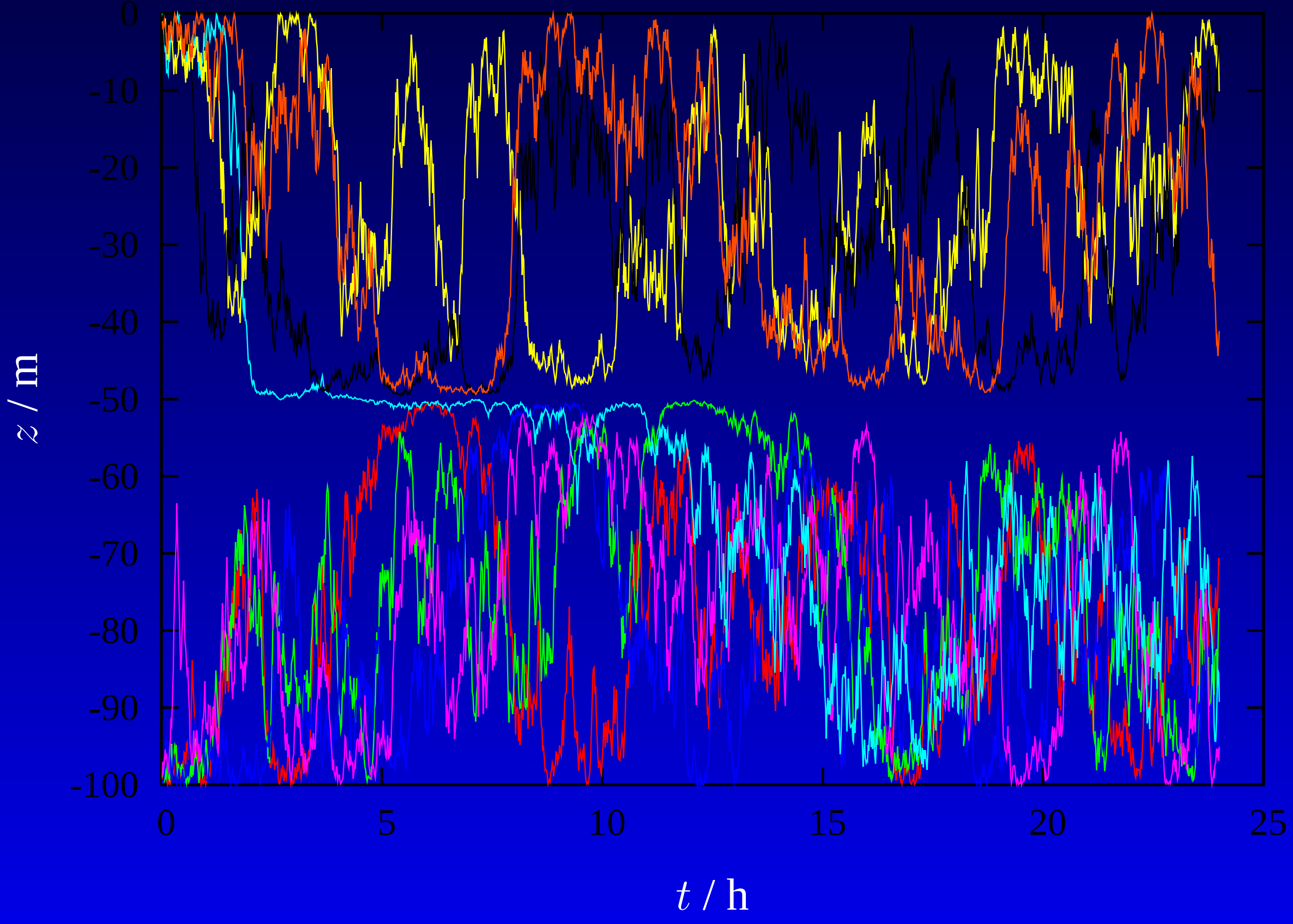




Homogeneous concentration

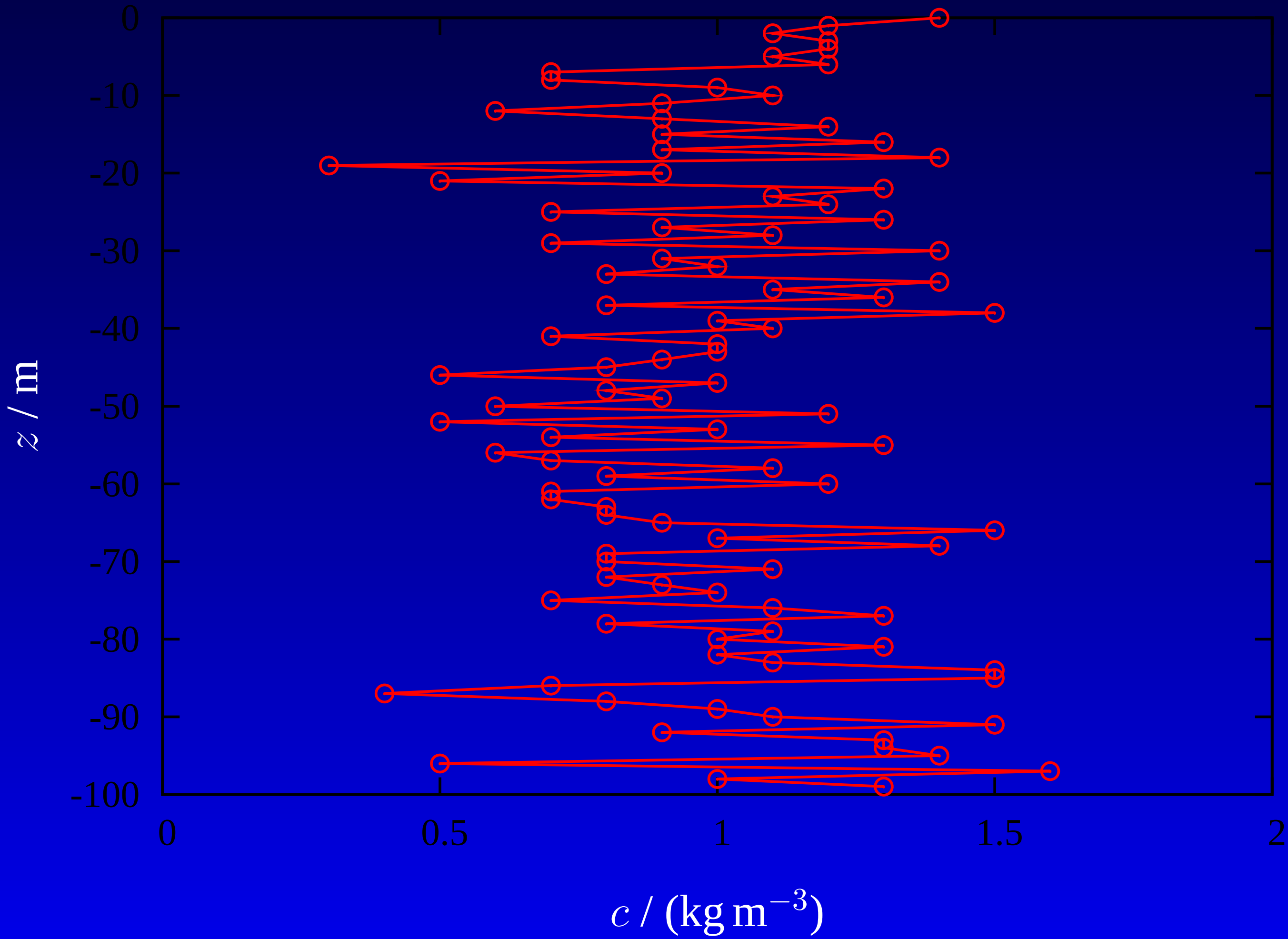
Some trajectories

Starts at: z=-0.1; -0.2; -0.3; -0.4; -99.6; -99.7; -99.8; -99.9 m



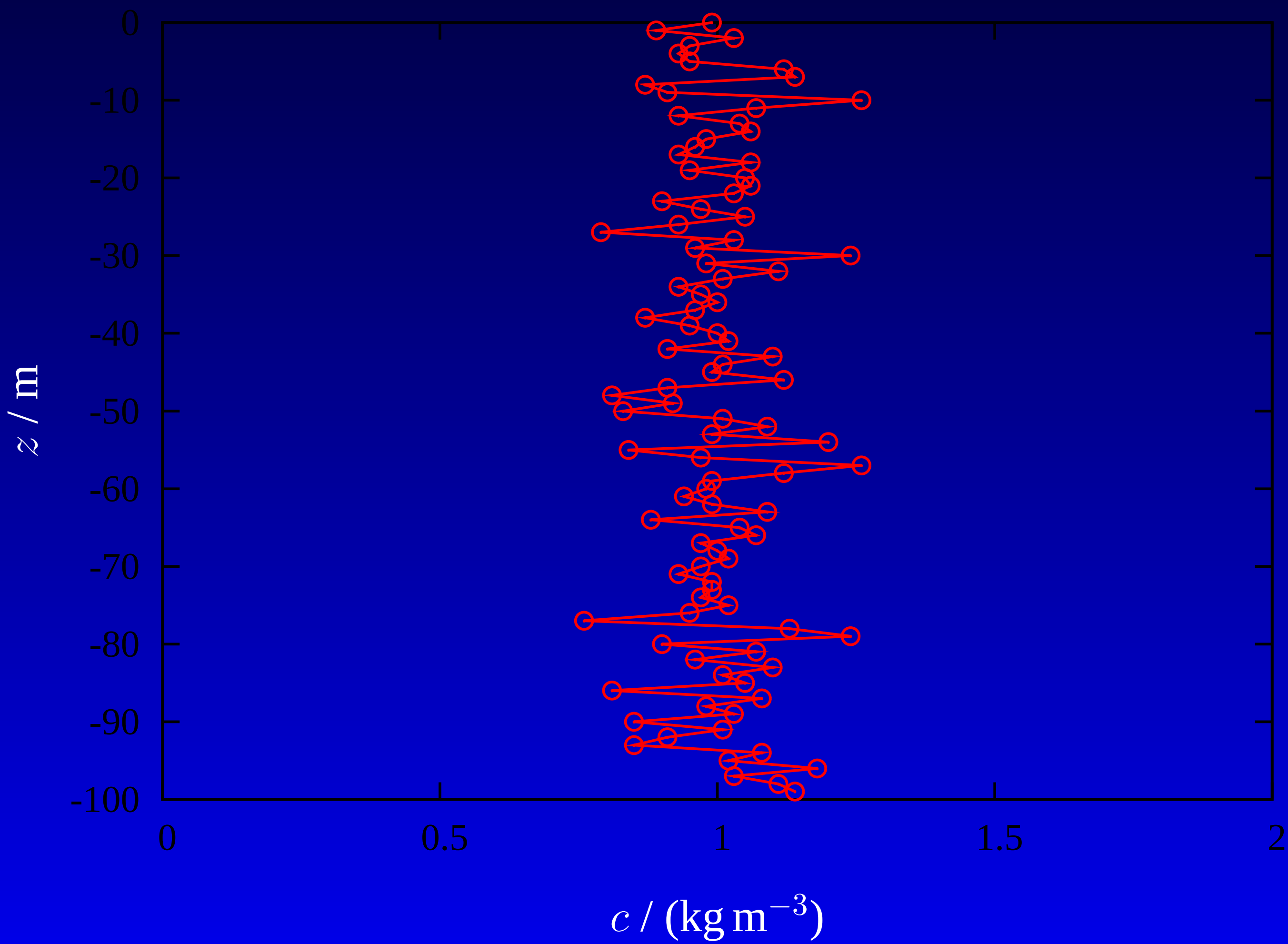
Homogeneous concentration

Concentrations for $N = 1000$ particles at t=24 h



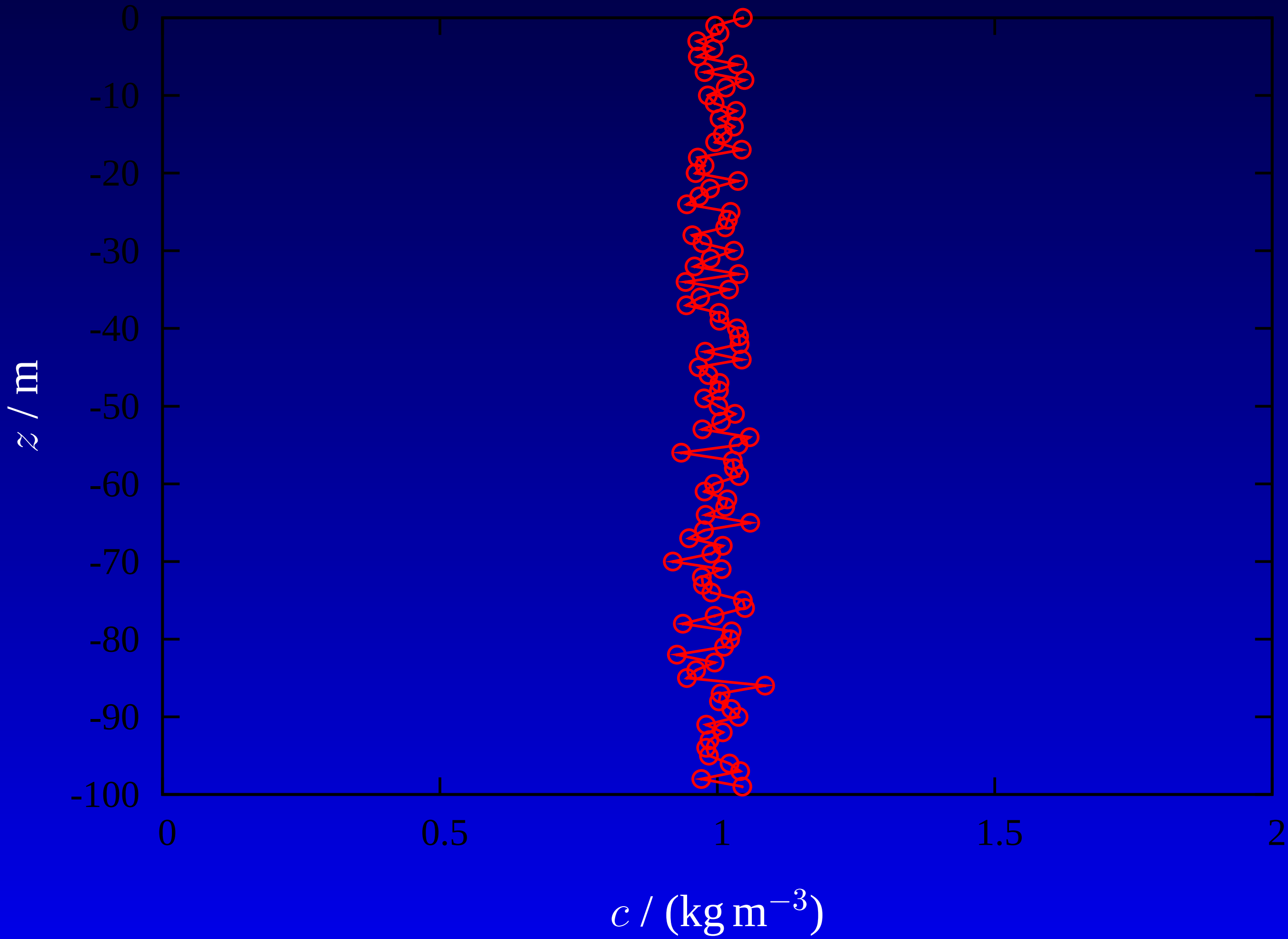
Homogeneous concentration

Concentrations for $N = 10000$ particles at t=24 h



Homogeneous concentration

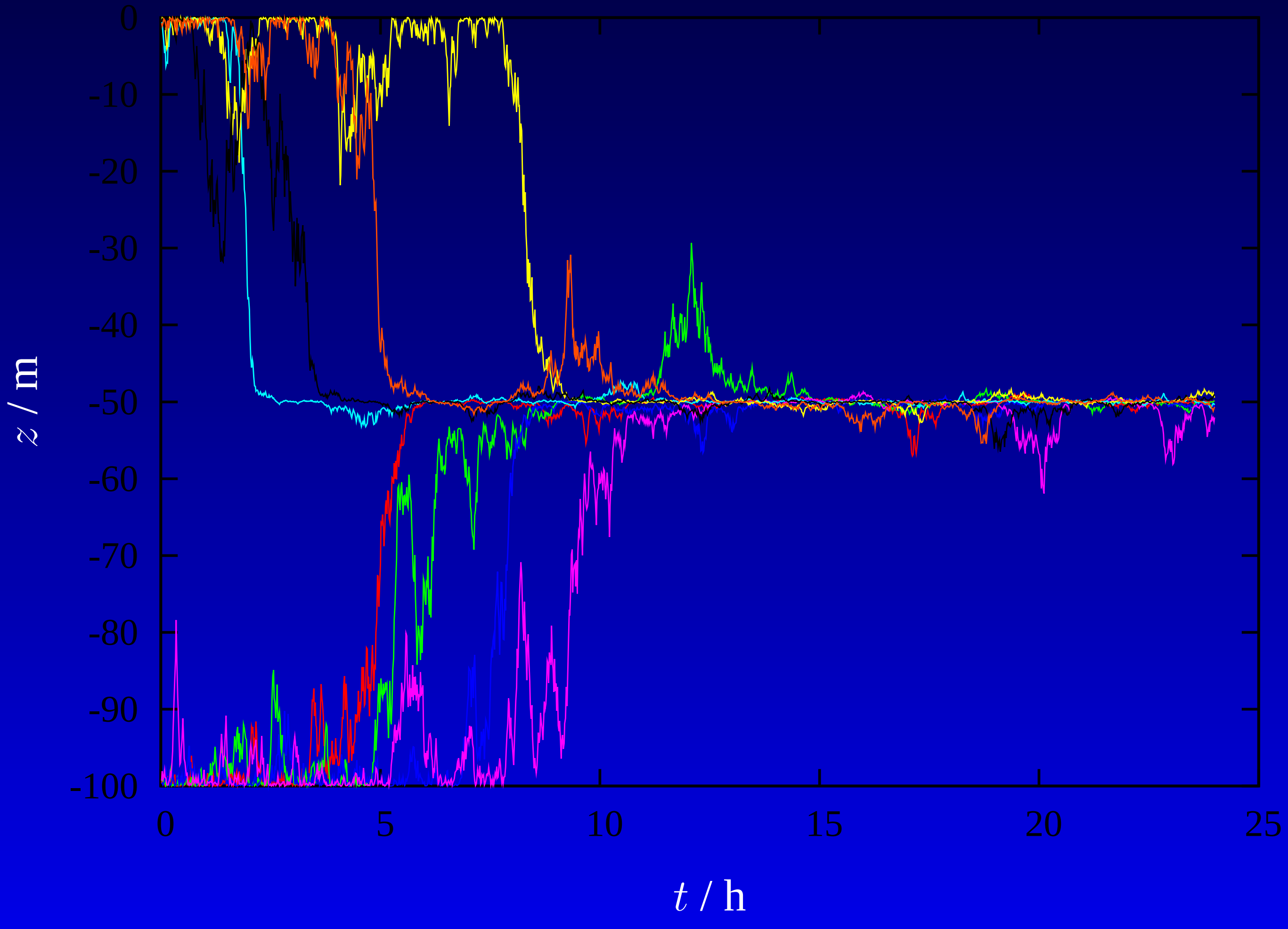
Concentrations for $N = 100000$ particles at t=24 h



Homogeneous concentration

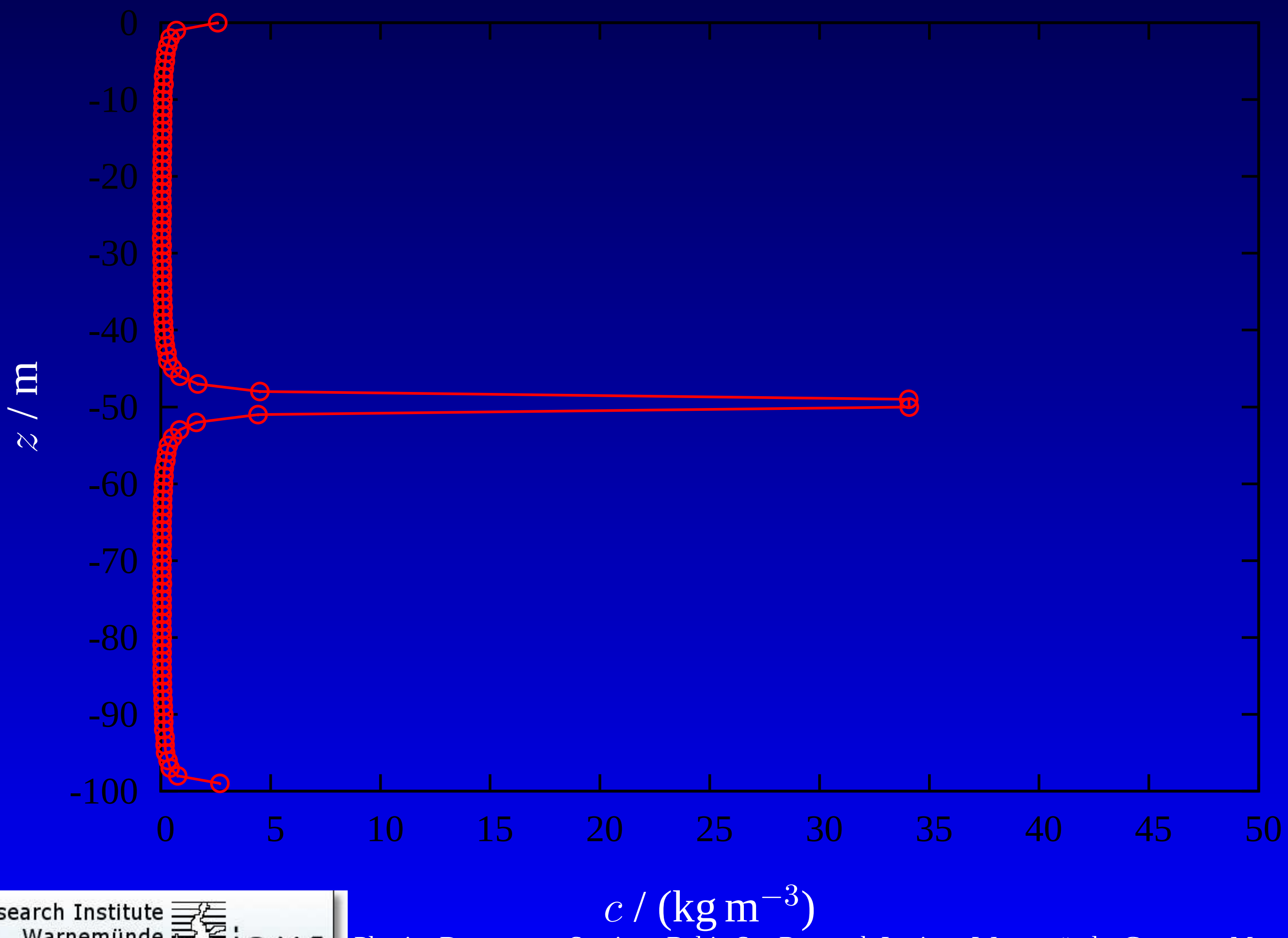
Some trajectories (inconsistent random walk)

Starts at: z=-0.1; -0.2; -0.3; -0.4; -99.6; -99.7; -99.8; -99.9 m



Homogeneous concentration

Concentrations for $N = 100000$ particles at t=24 h (inconsistent random walk)



Homogeneous concentration

Reason for failure of naive random walk

Naive random walk is consistent with (Visser [1997])

$$\partial_t C - \partial_{zz}\left(\nu_t C\right) = 0 \tag{3}$$

which is equivalent to

$$\partial_t C - \partial_z(\partial_z \nu_t C) - \partial_z\left(\nu_t \partial_z C\right) = 0. \tag{4}$$

Thus, an advection with the advective velocity $-\partial_z \nu_t$ (against the viscosity gradient) is performed.

Rouse profile

Analytical Problem

- Constant settling velocity w_c
- Parabolic eddy diffusivity ν_t
- Reflective bottom and surface
- Steady-state solution

$$\partial_t C + \partial_z \left(w_c C - \nu_t \partial_z C \right) = 0, \tag{5}$$

with

$$\nu_t = \kappa u_* (-z) \frac{D + z_0 + z}{D + z_0} . \tag{6}$$

Rouse profile

Analytical Solution

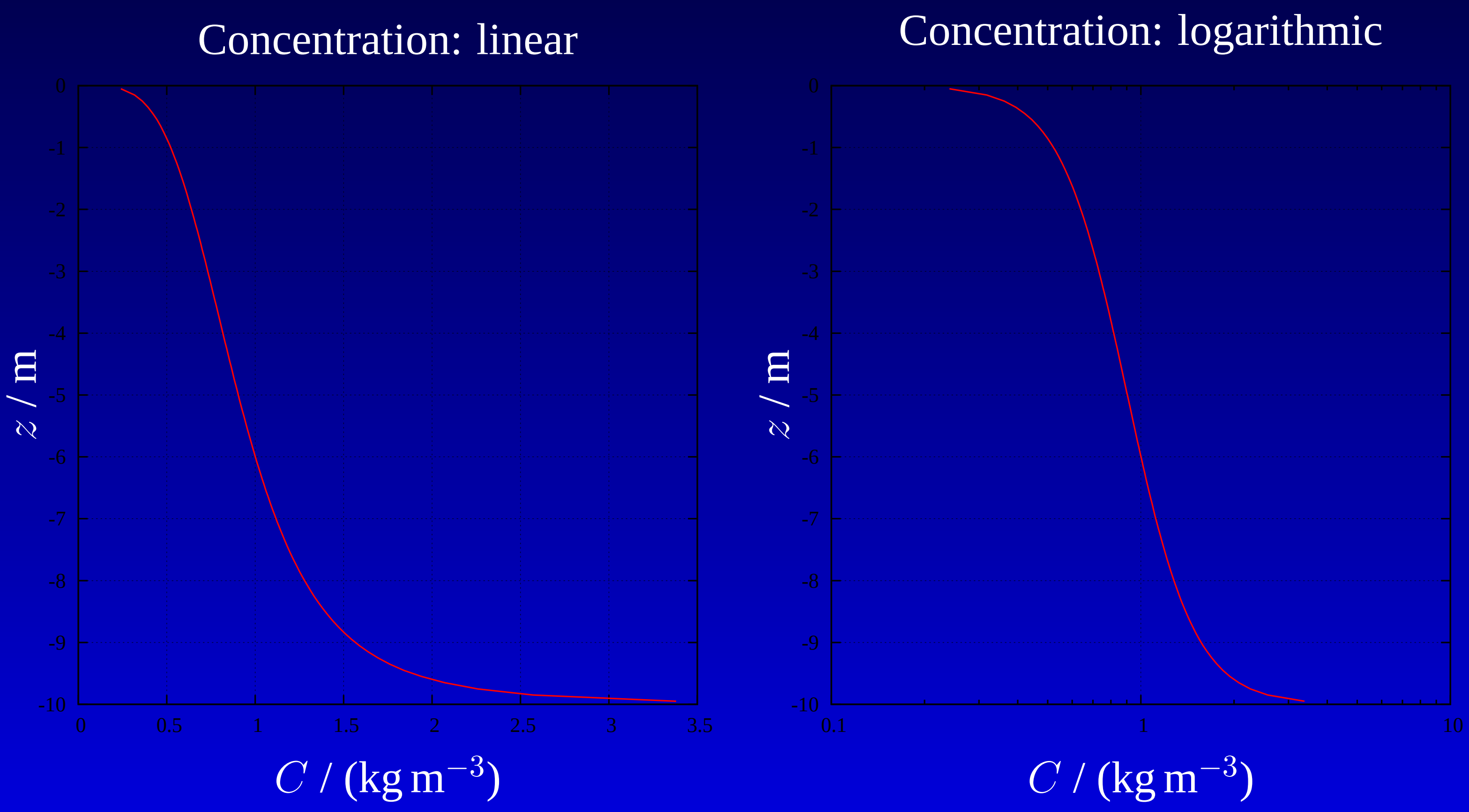
$$\frac{C}{C_0} = \left(\frac{-z}{D + z_0 + z}\right)^{-w_c/(\kappa u_*)} \tag{7}$$

Rouse number:

$$R = \frac{-w_c}{u_*}. \tag{8}$$

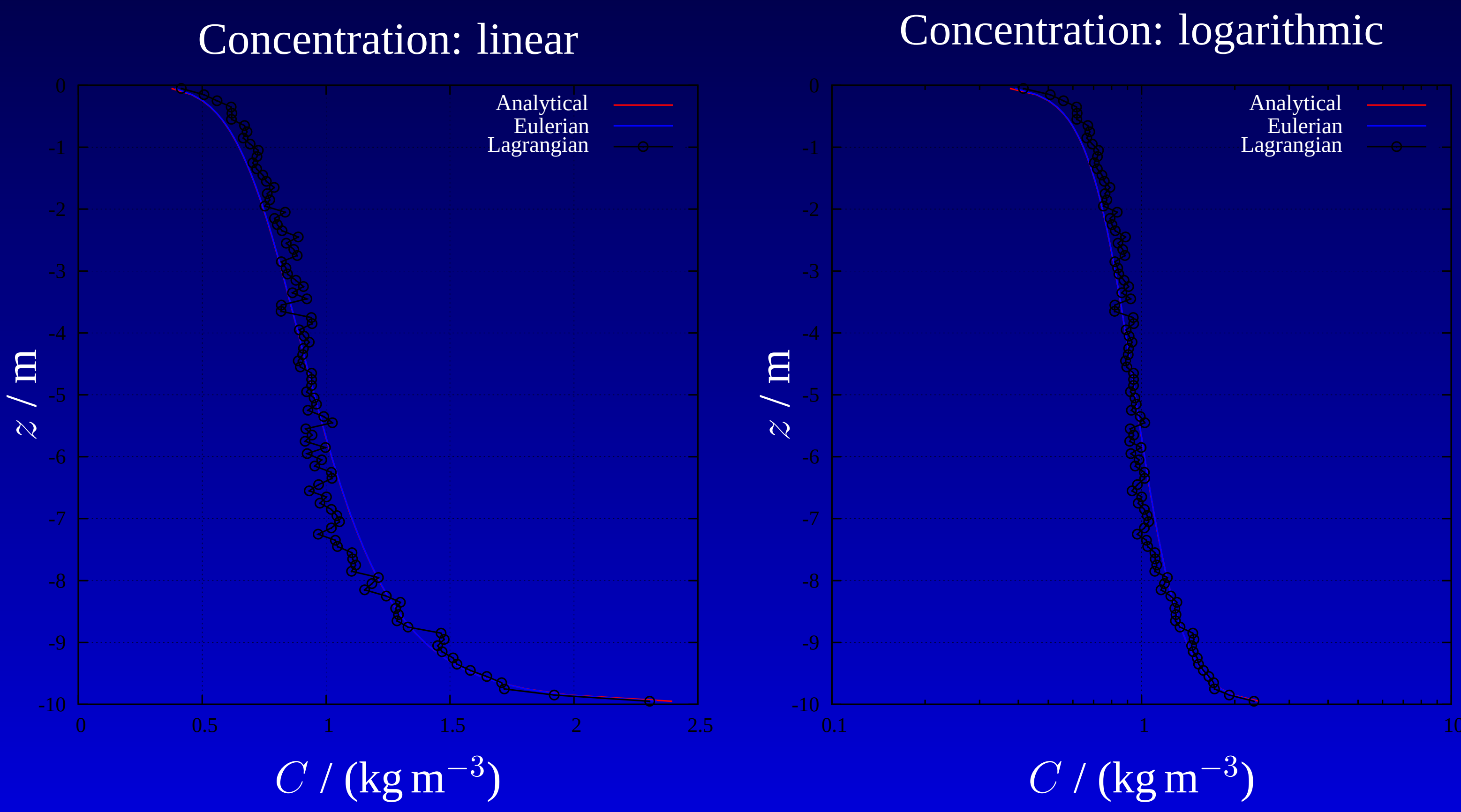
Rouse profile

Analytical Solution



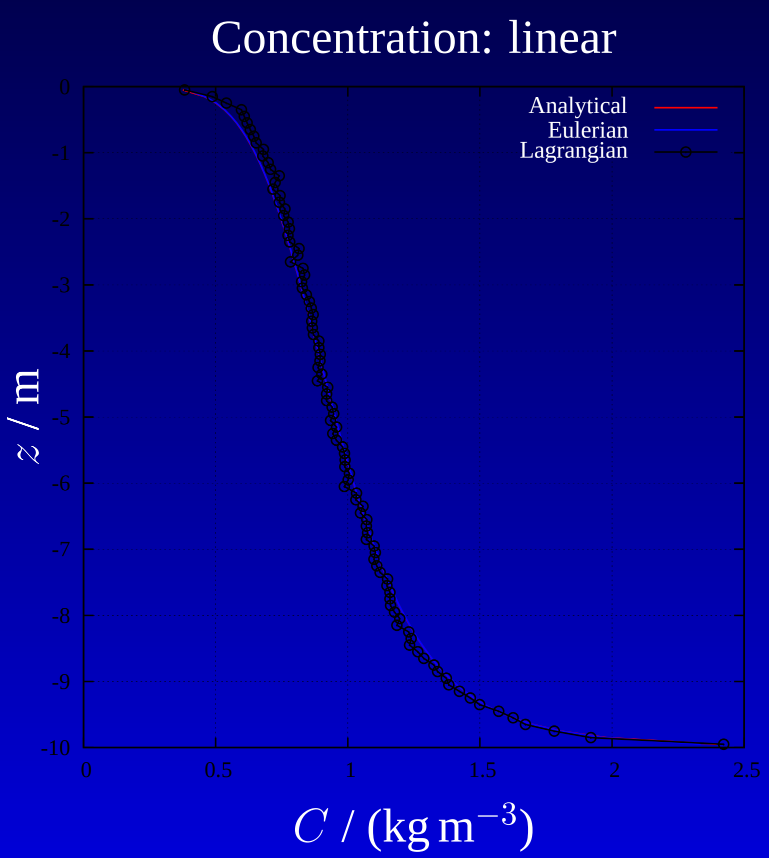
Rouse profile

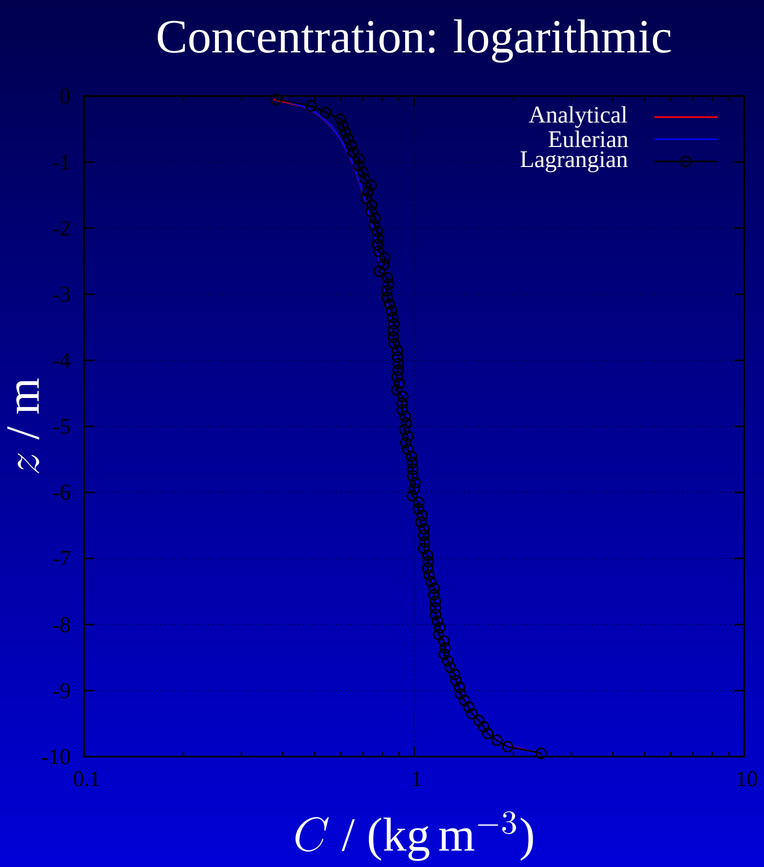
Numerical Solution for N = 1000 Particles



Rouse profile

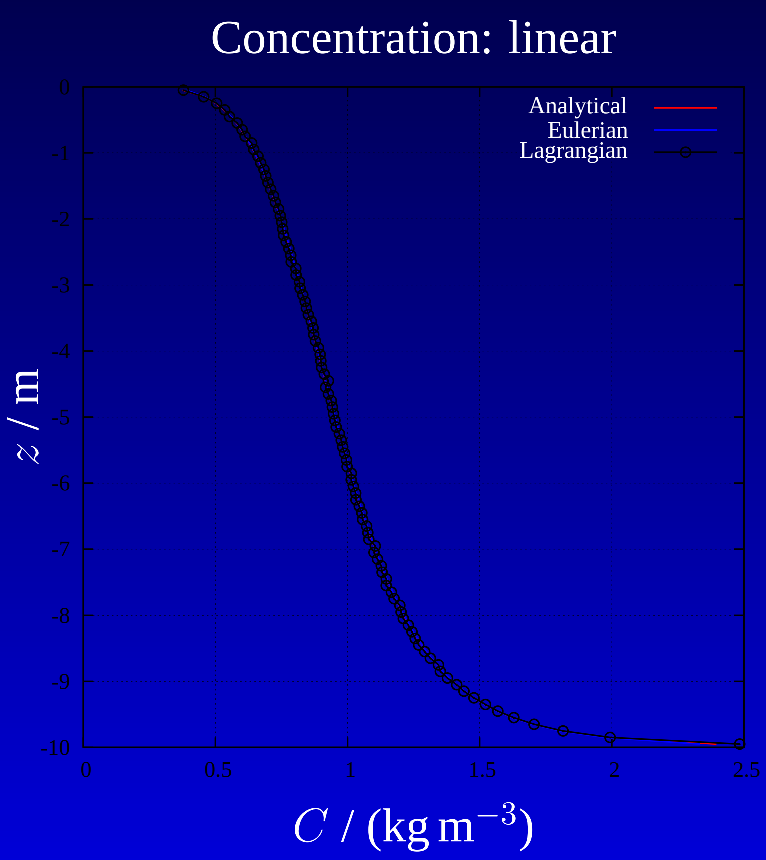
Numerical Solution for N = 10000 Particles

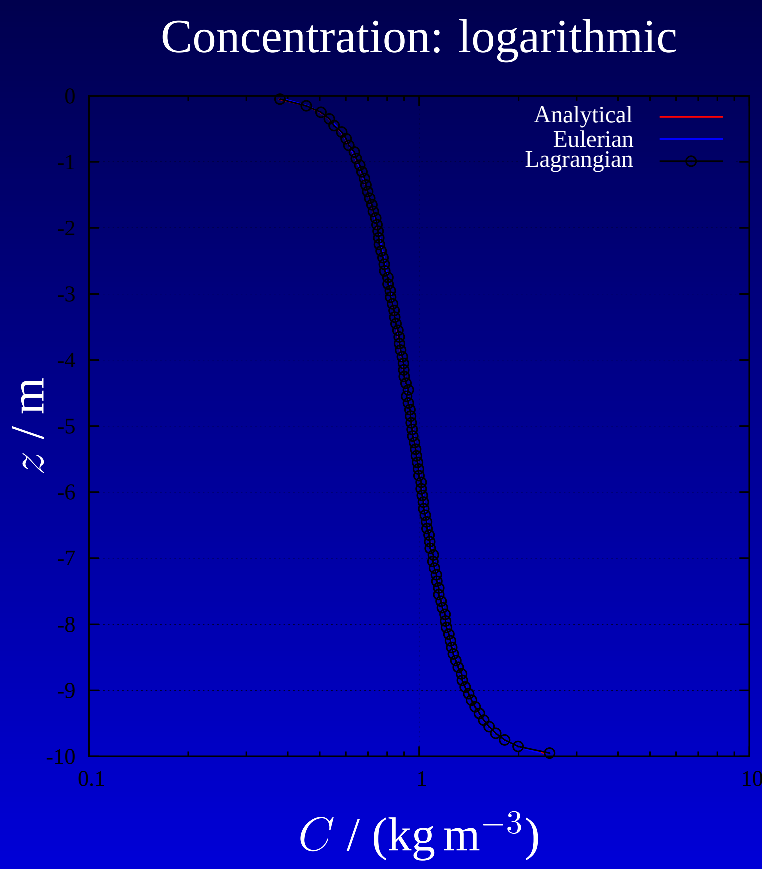




Rouse profile

Numerical Solution for N = 100000 Particles





Rouse profile

Error with respect to analytical solution (standard deviation)

Lagrangian scheme

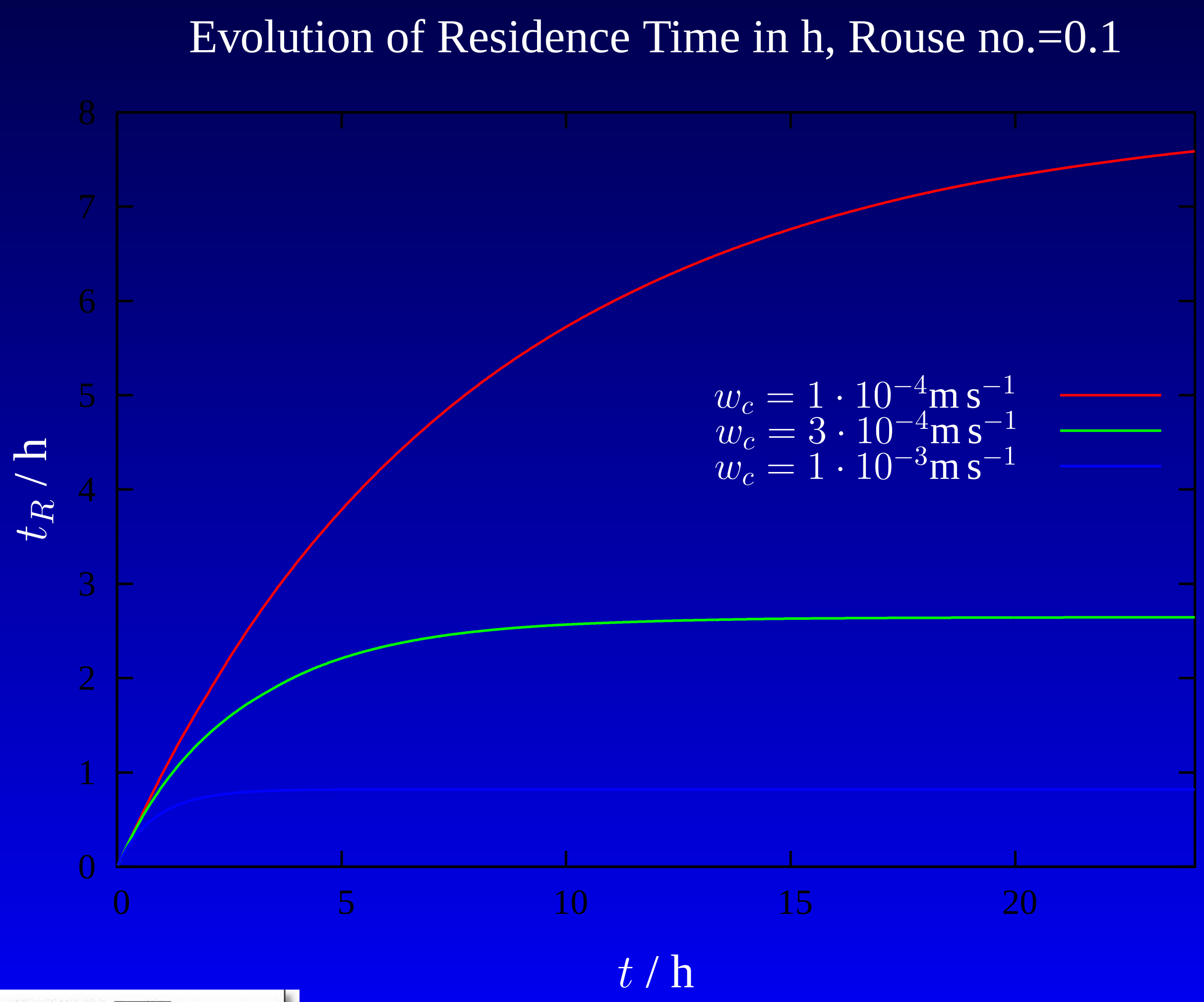
N	10^3	10^4	10^5
σ	0.059	0.023	0.010

Eulerian scheme

Advection scheme	upwind	TVD
σ	0.011	0.0064

Residence time

Residence time in water column (above $z = -9$ m)
Start at $z = 5$ m, $N = 10000$



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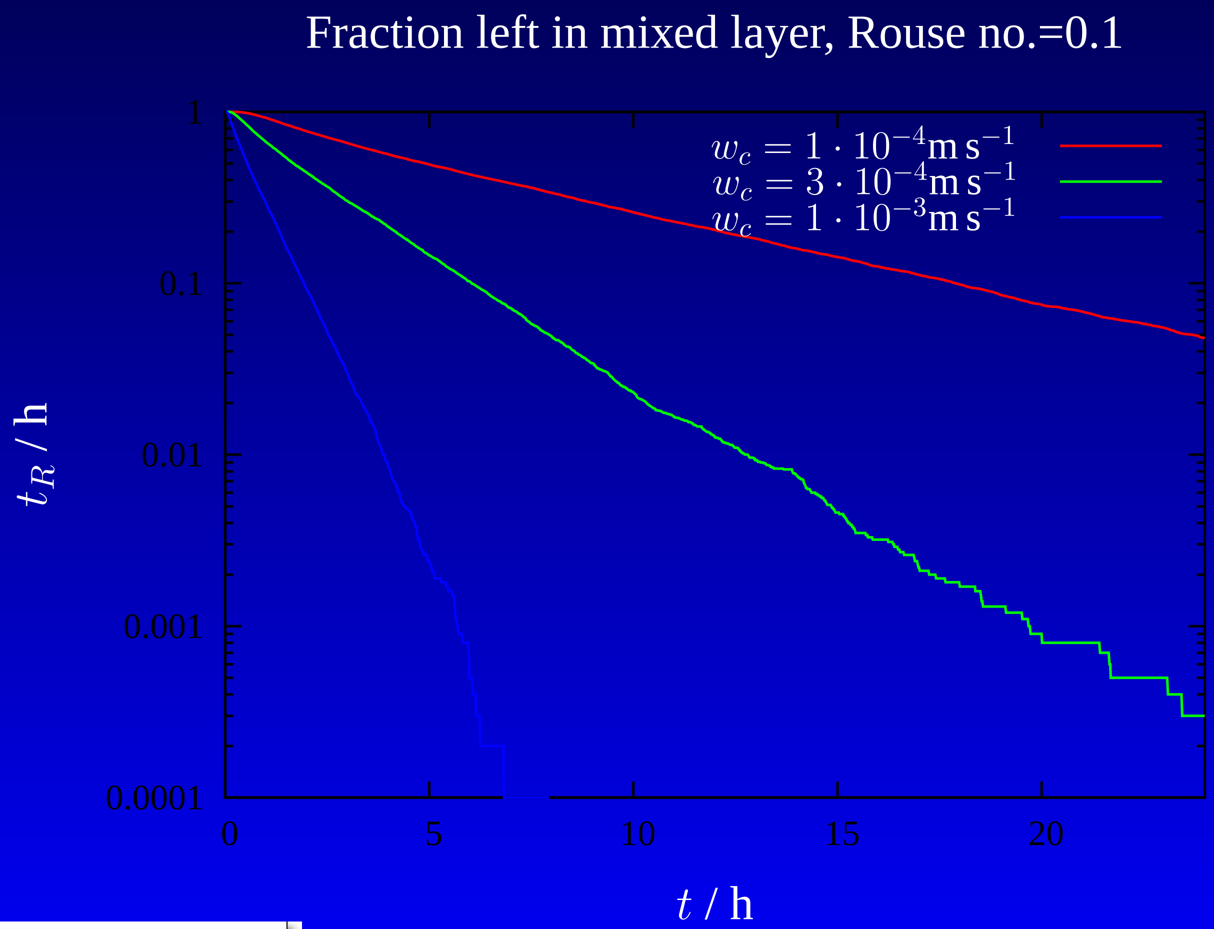


Mussel filtration

Benthic filter feeders as sinks for suspended particulate matter

Eulerian formulation of boundary condition:

$$\nu_t \partial_z C\Big|_{z=-H} = w_f C\Big|_{z=-H}, \qquad w_f = N_m V_f \qquad (9)$$

w_f: filtration velocity

N_m: number of mussels per m^2 (e.g. 1000)

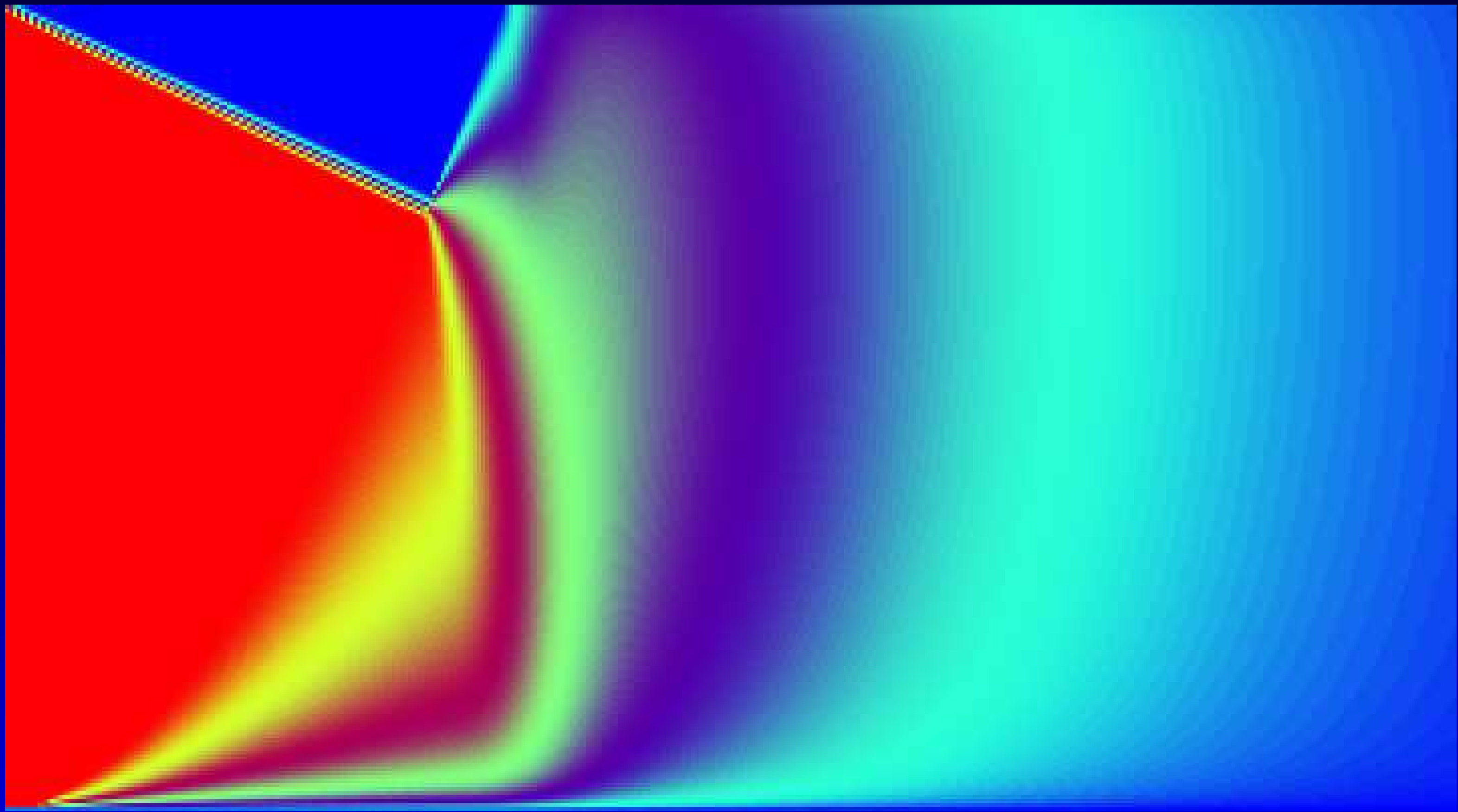
V_f: filtration volume per mussel per hour (e.g. 18 l)

Lagrangian formulation of boundary condition:

At each time step take out all particles which are below $z = -H + \Delta t w_f$.

Mussel filtration

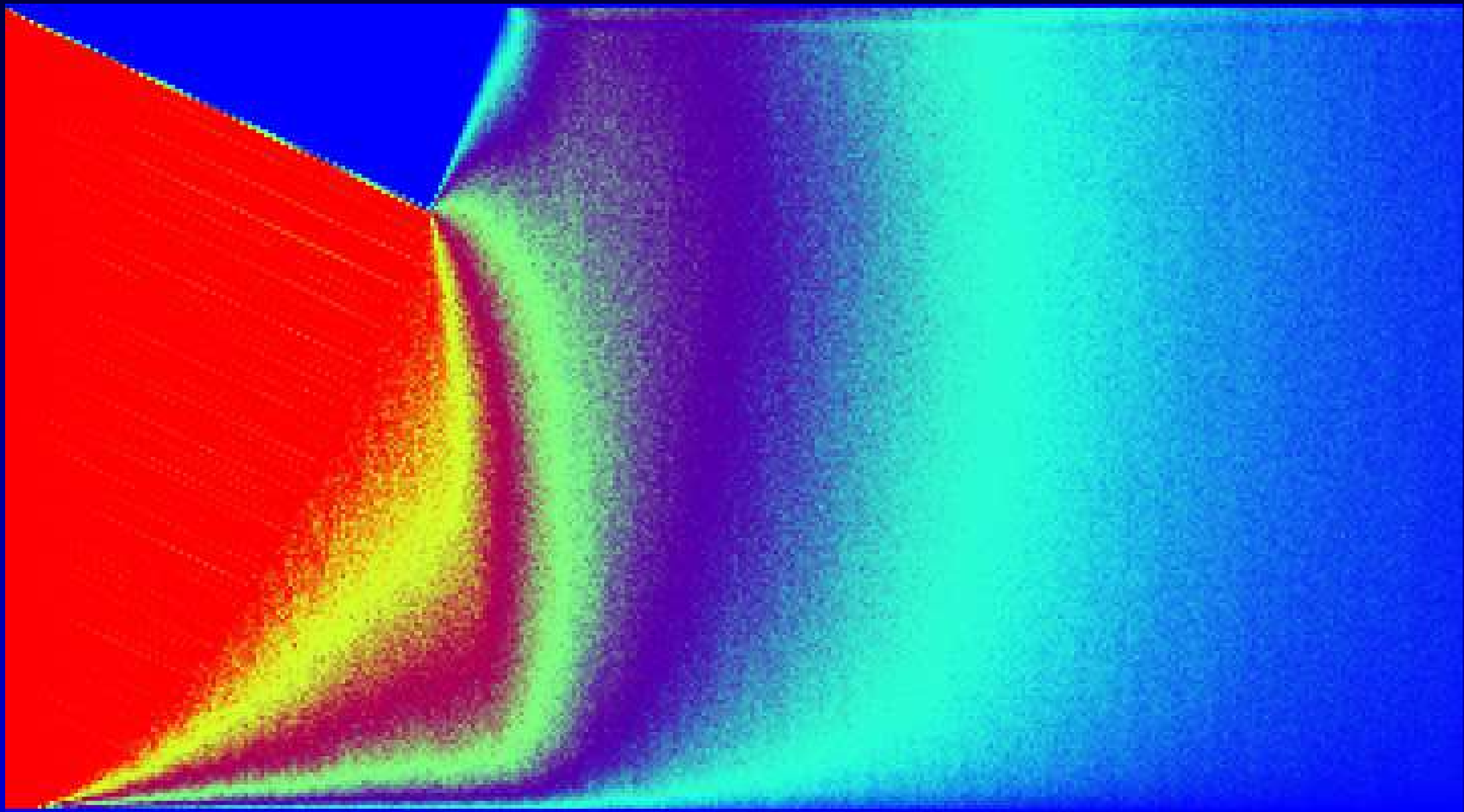
Profiles of suspended matter: Euler



(x-axis: 0h $\leq t \leq$ 12 h, z-axis: -10 m $\leq z \leq$ 0 m, contours: $0 \leq C \leq 1$)

Mussel filtration

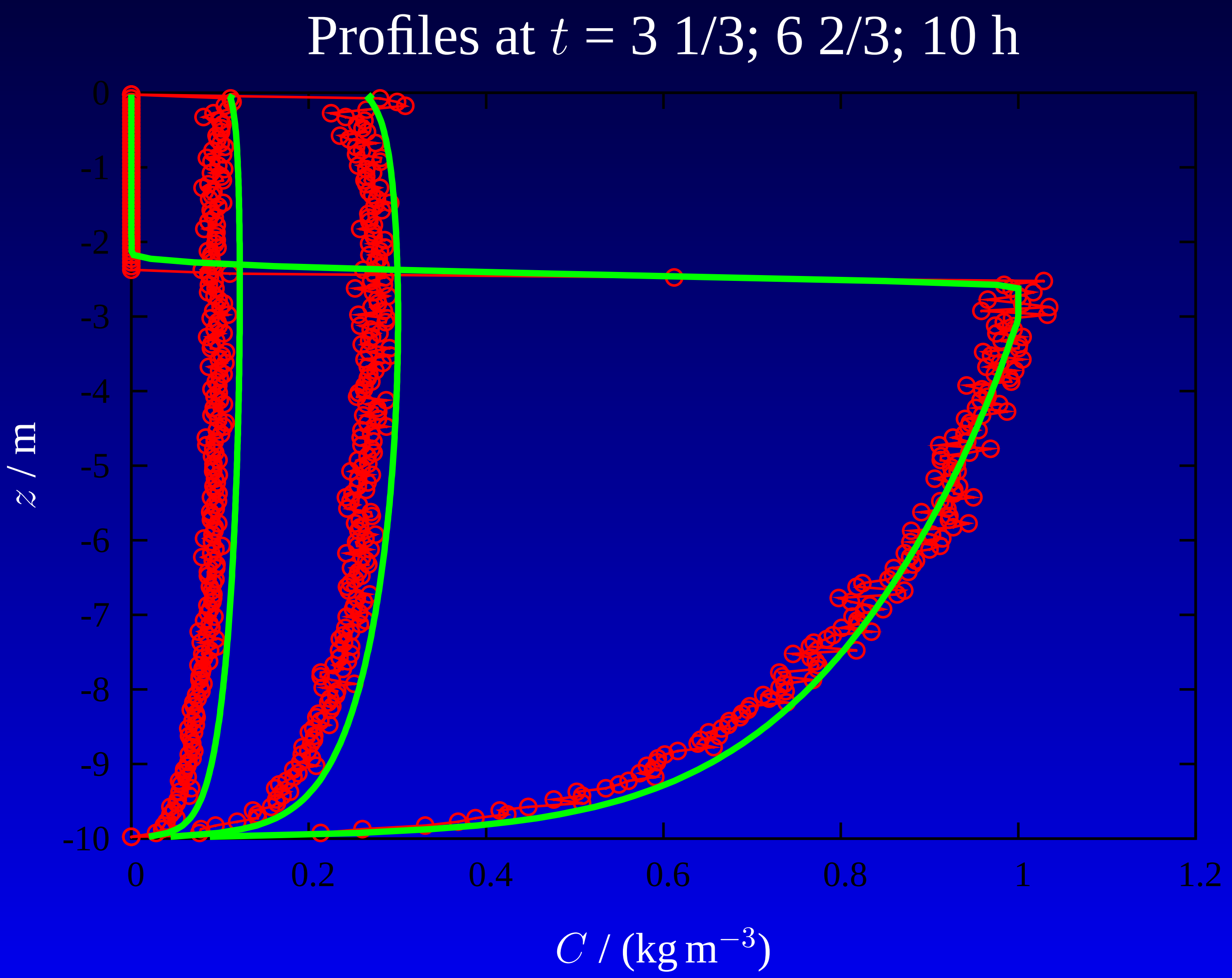
Profiles of suspended matter: Lagrange



(x-axis: 0h $\leq t \leq$ 12 h, z-axis: -10 m $\leq z \leq$ 0 m, contours: $0 \leq C \leq 1$)

Mussel filtration

Profiles of suspended matter: Euler vs. Lagrange



Project: AlgaLag

A Lagrangian Study of Algal Bloom in the Ocean Mixed Layer (AlgaLag)

- German-Japanese Cooperation
- DFG-funding 2004-2007 (Travel expenses to Japan)
- Partner: Prof. Hidekatsu Yamazaki (Tokyo University of Fisheries)
- Objectives:
  - State-of-the-art Lagrangian model into GOTM
  - Improve random walk models for turbulence interaction
  - Lagrangian model for ecosystem models
  - Include memory effects into Lagrangian model
  - Improved parameterisations for Eulerian models