

Lagrangian Particle Modelling in Inhomogeneous Turbulent Flow

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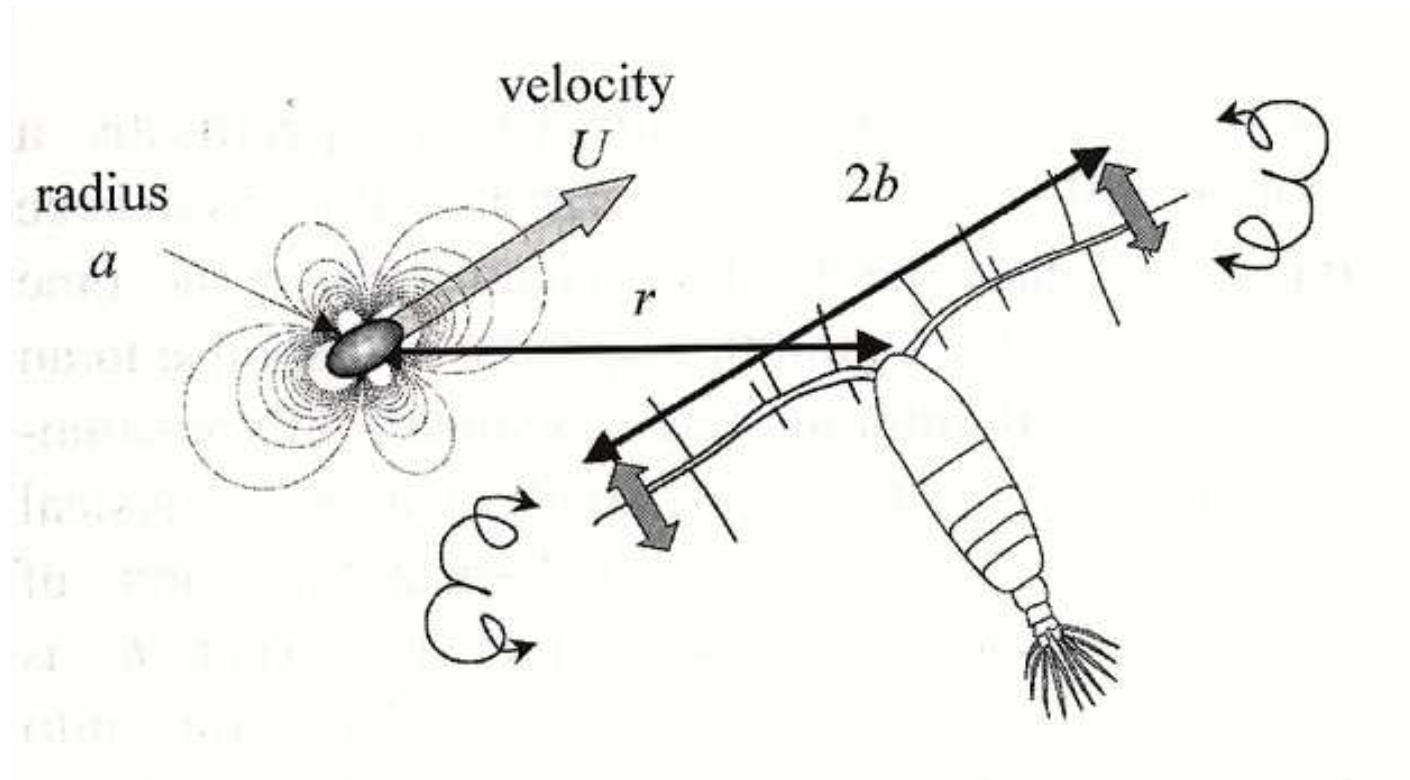
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Contents

- Random walk consistent with diffusion
- Basic test: homogeneous concentration
- Less basic test: Rouse profile
- Advanced test: mussel filtration
- Project sketch: AlgaLag
- First results for AlgaLag: Photo response model
- No conclusions yet

Motivation

Complex *particle-particle* interaction:

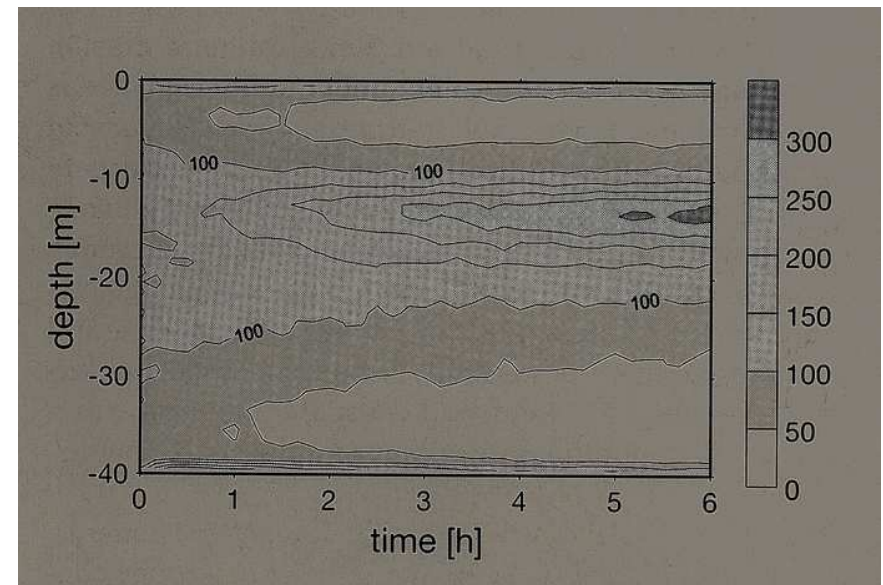
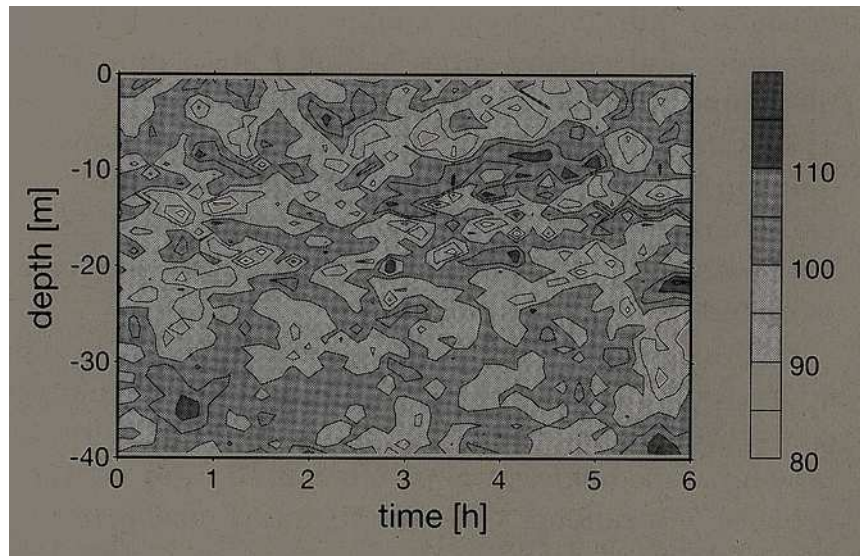


Any chance to model this for a water column ?

Reference: Visser and Stips [2002]

Motivation

Consistent (left) and inconsistent (right) random walk:



(Based on viscosity profile with surface and bottom mixed layer and minimum at $z = -10$ m)

Random walk

Random walk consistent with diffusion equation:

$$\partial_t C - \partial_z (\nu_t \partial_z C) = 0. \quad (1)$$

$$z_i^{n+1} = z_i^n + \partial_z \nu_t(z_i^n) \Delta t + R \left\{ 2r^{-1} \nu_t(z_i^n) + \frac{1}{2} \partial_z \nu_t(z_i^n) \Delta t \right\} \Delta t \quad (2)$$

R : random process with $\langle R \rangle = 0$ and $\langle R^2 \rangle = r$.

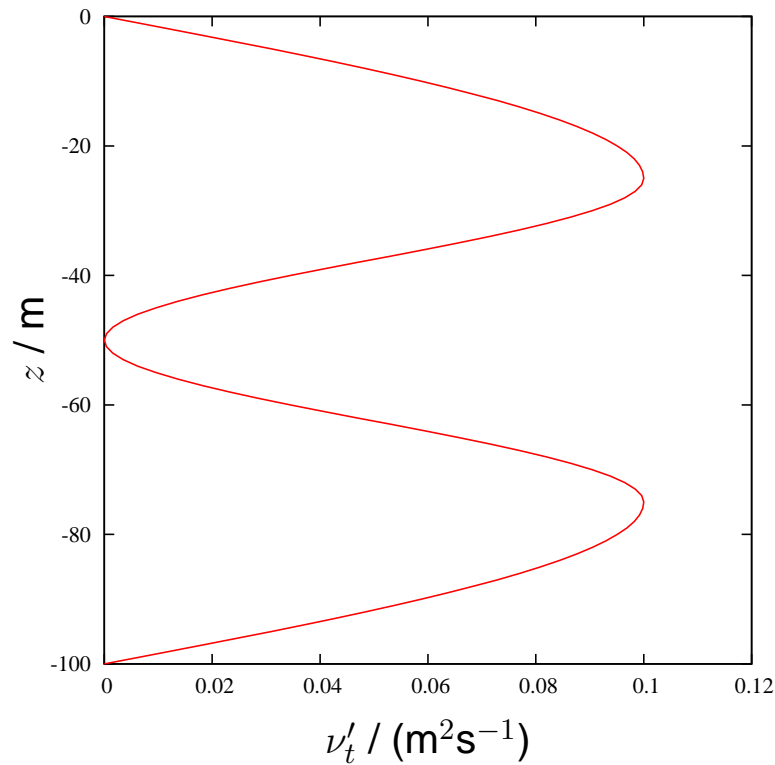
z_i^n : vertical position of particle i at time step n .

Reference: Visser [1997]

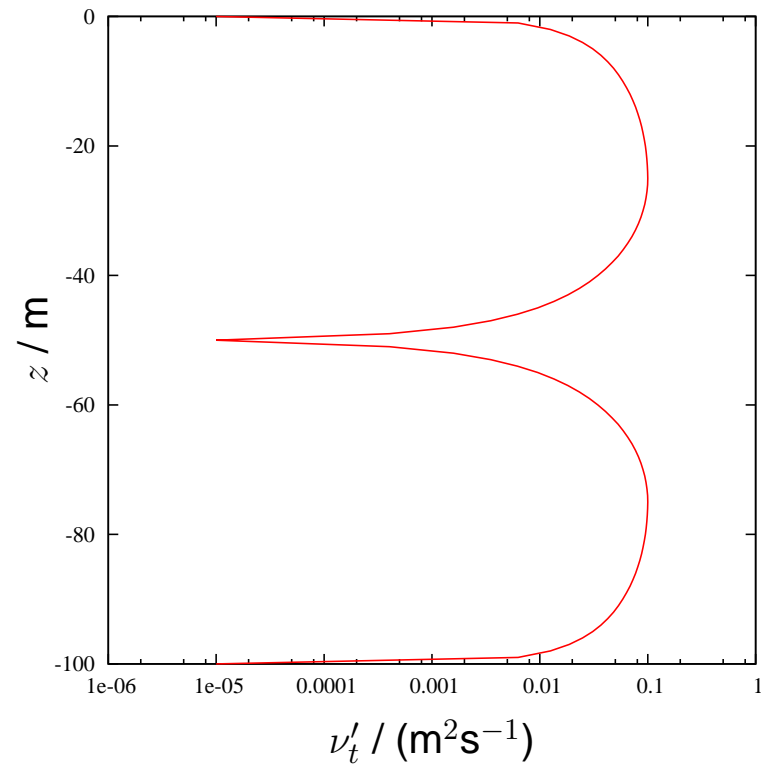
Homogeneous concentration

Prescribed diffusivity profile

Diffusivity: linear



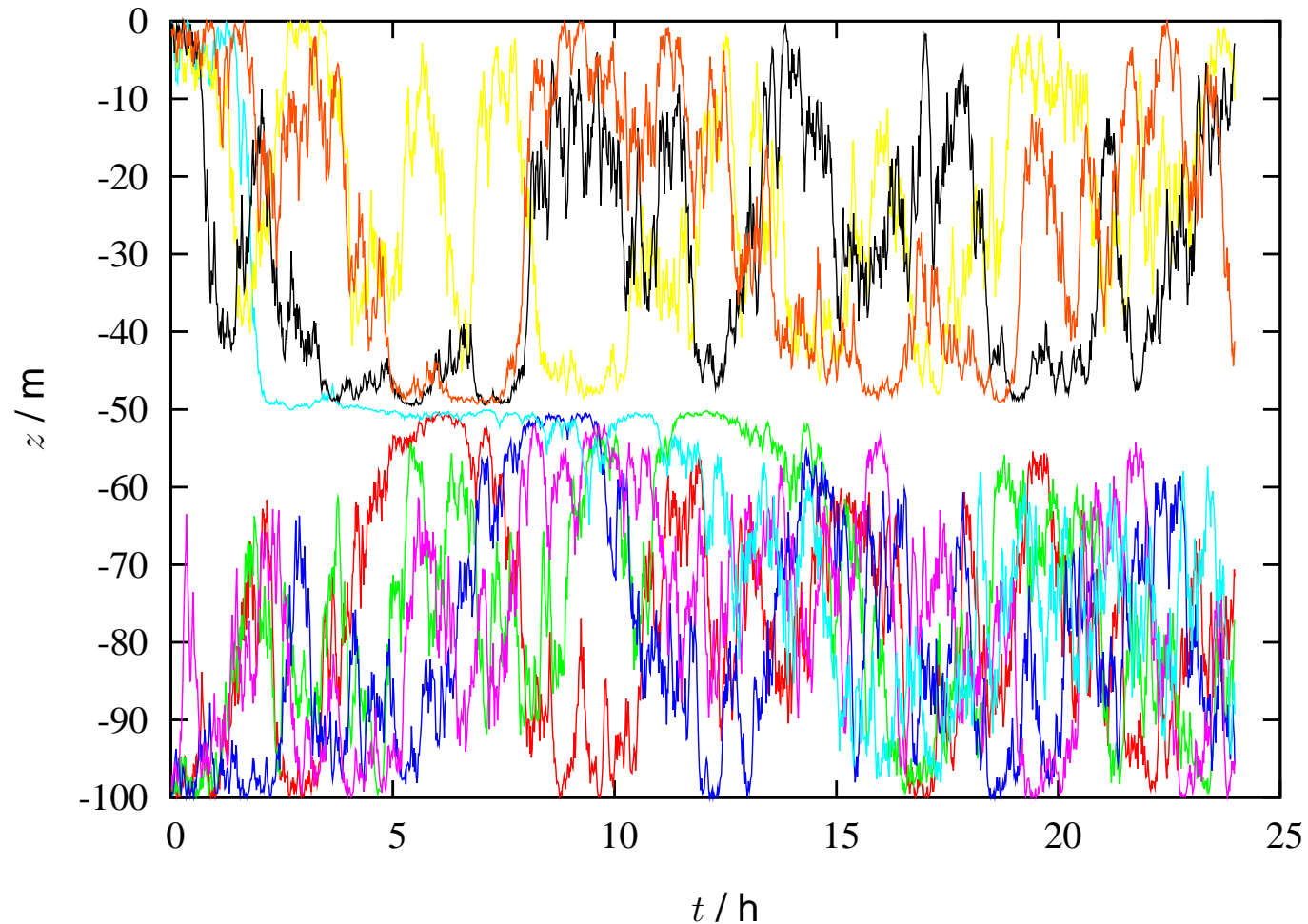
Diffusivity: logarithmic



Homogeneous concentration

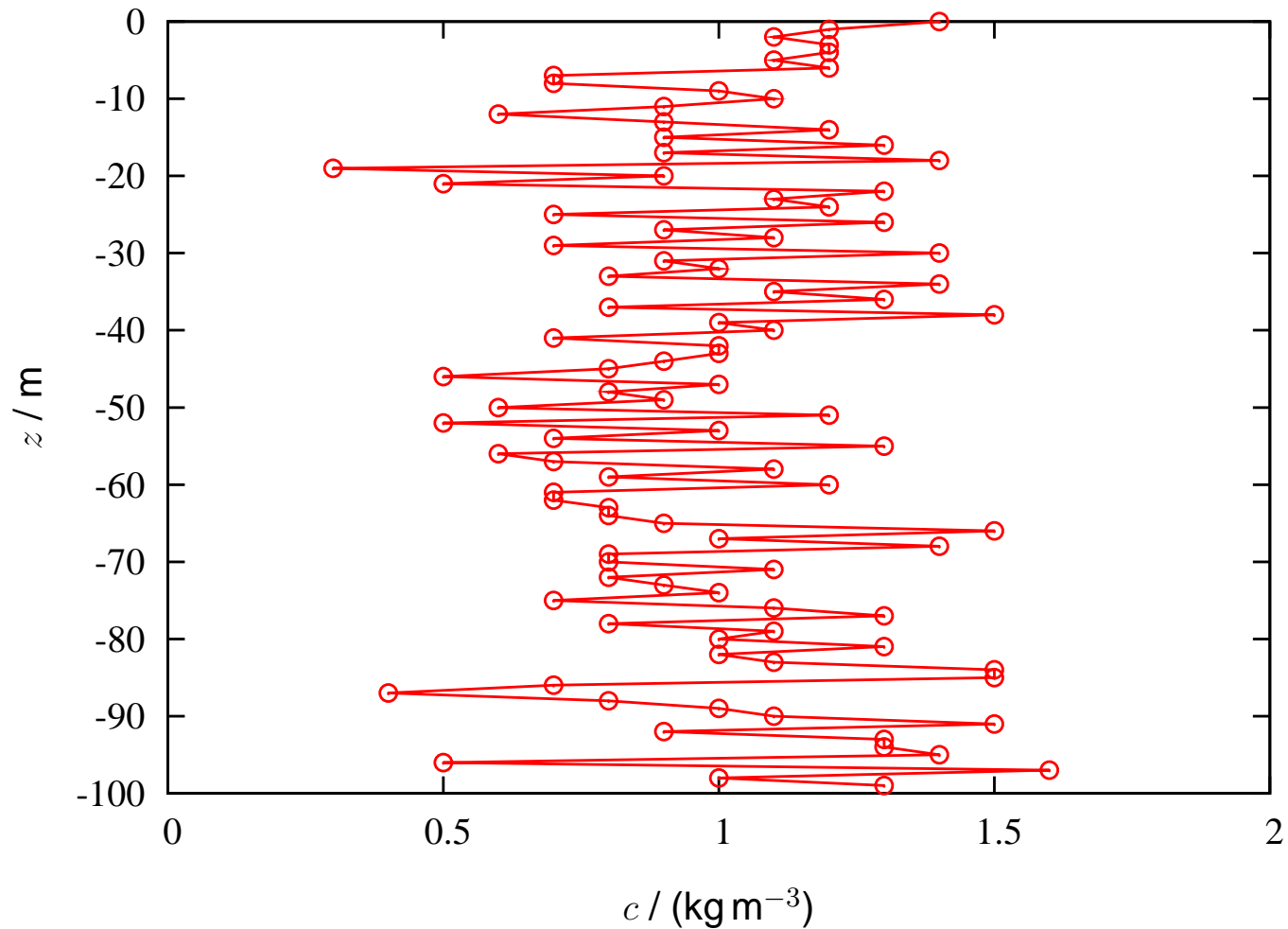
Some trajectories

Starts at: $z=-0.1$; -0.2 ; -0.3 ; -0.4 ; -99.6 ; -99.7 ; -99.8 ; -99.9 m



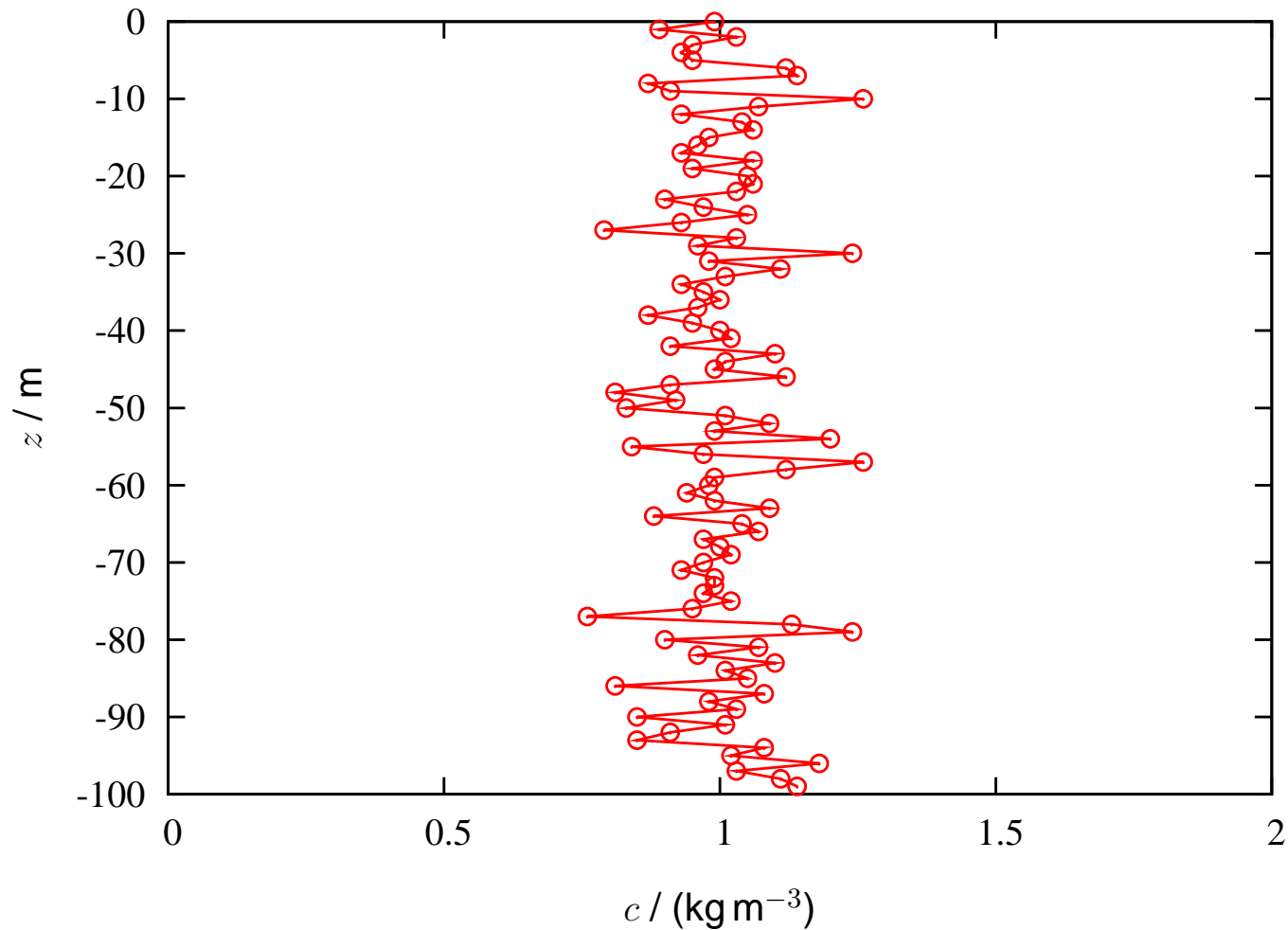
Homogeneous concentration

Concentrations for $N = 1000$ particles at $t=24$ h



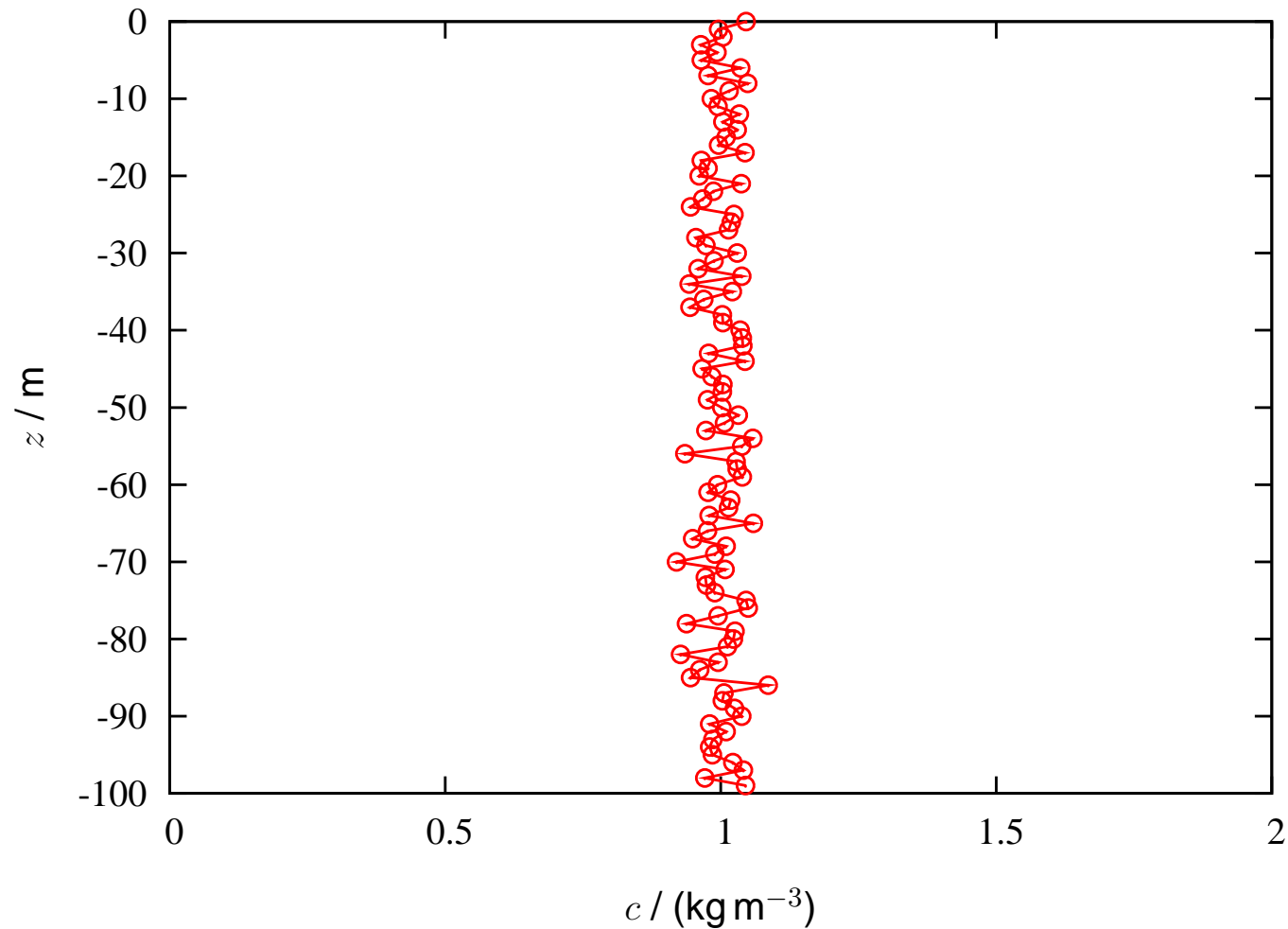
Homogeneous concentration

Concentrations for $N = 10000$ particles at $t=24$ h



Homogeneous concentration

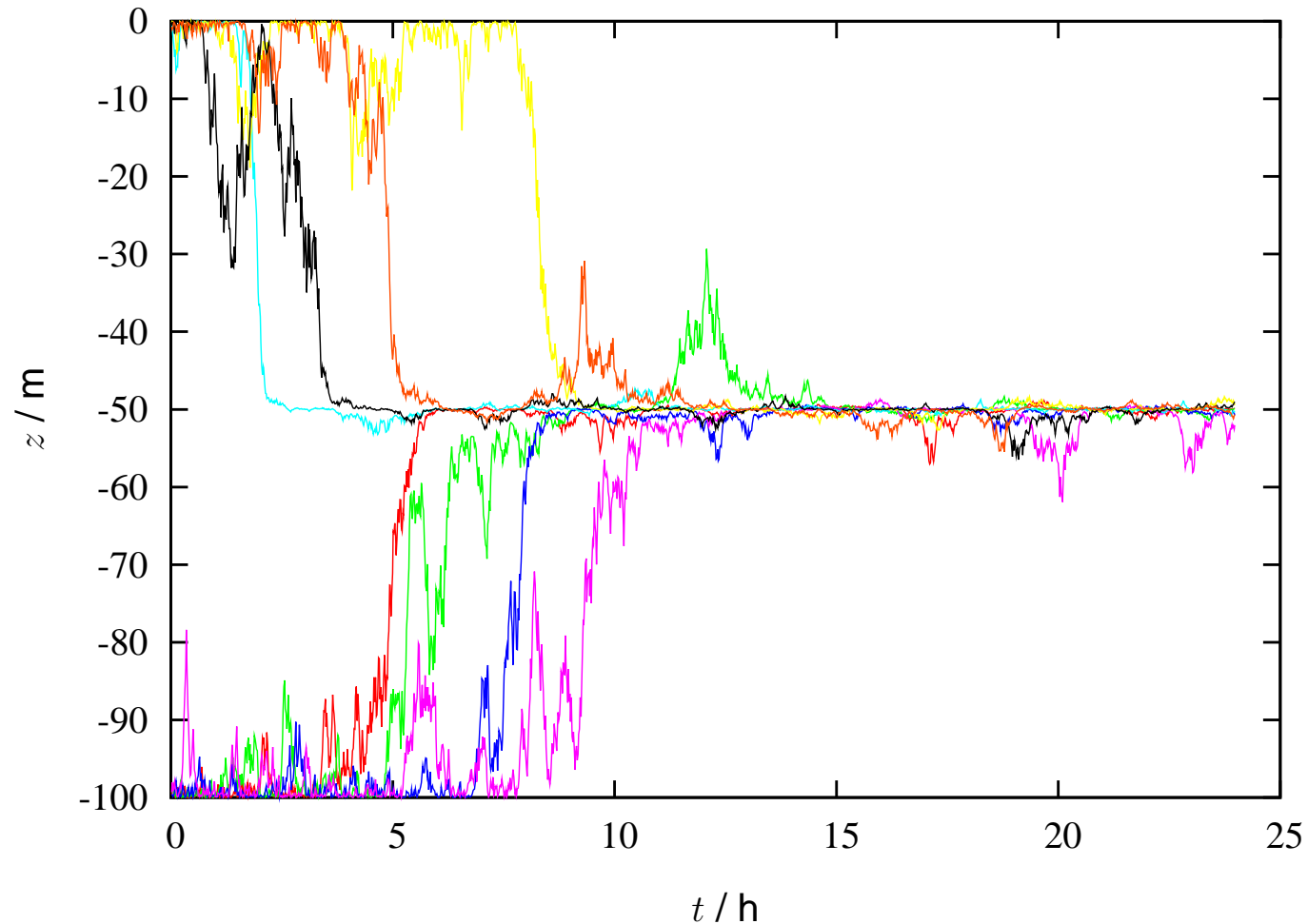
Concentrations for $N = 100000$ particles at $t=24$ h



Homogeneous concentration

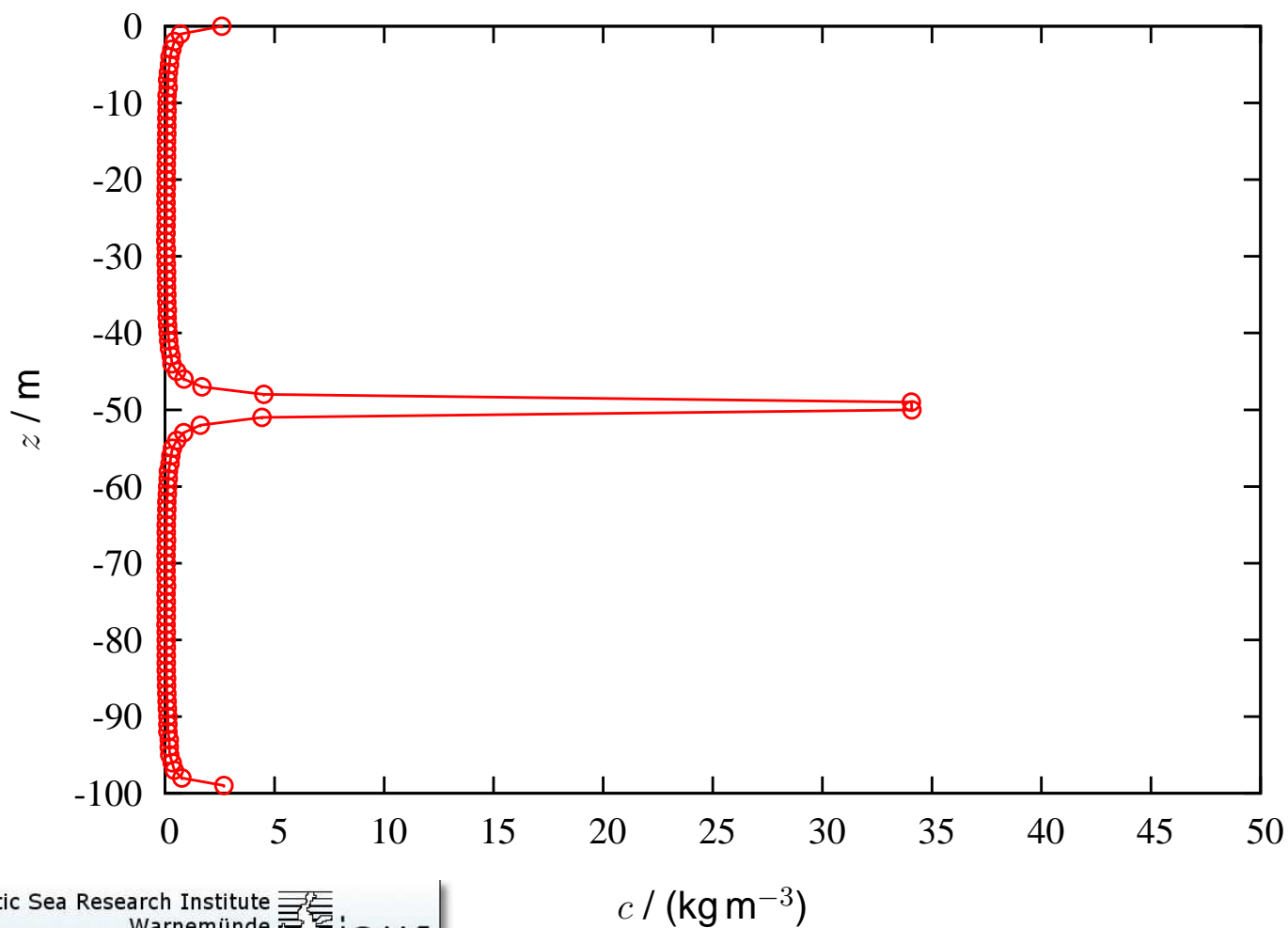
Some trajectories (inconsistent random walk)

Starts at: $z=-0.1$; -0.2 ; -0.3 ; -0.4 ; -99.6 ; -99.7 ; -99.8 ; -99.9 m



Homogeneous concentration

Concentrations for $N = 100000$ particles at $t=24$ h
(inconsistent random walk)



Homogeneous concentration

Reason for failure of naive random walk

Naive random walk is consistent with (Visser [1997])

$$\partial_t C - \partial_{zz} (\nu_t C) = 0 \quad (3)$$

which is equivalent to

$$\partial_t C - \partial_z (\partial_z \nu_t C) - \partial_z (\nu_t \partial_z C) = 0. \quad (4)$$

Thus, an advection with the advective velocity $-\partial_z \nu_t$ (against the viscosity gradient) is performed.

Rouse profile

Analytical Problem

- Constant settling velocity w_c
- Parabolic eddy diffusivity ν_t
- Reflective bottom and surface
- Steady-state solution

$$\partial_t C + \partial_z (w_c C - \nu_t \partial_z C) = 0, \quad (5)$$

with

$$\nu_t = \kappa u_* (-z) \frac{D + z_0 + z}{D + z_0}. \quad (6)$$

Rouse profile

Analytical Solution

$$\frac{C}{C_0} = \left(\frac{-z}{D + z_0 + z} \right)^{-w_c/(\kappa u_*)} \quad (7)$$

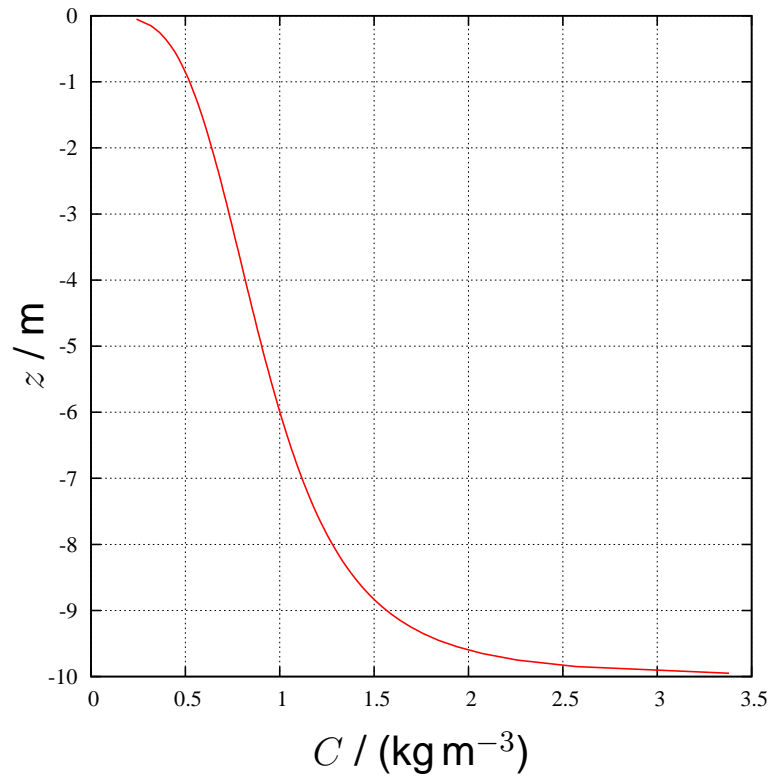
Rouse number:

$$R = \frac{-w_c}{u_*}. \quad (8)$$

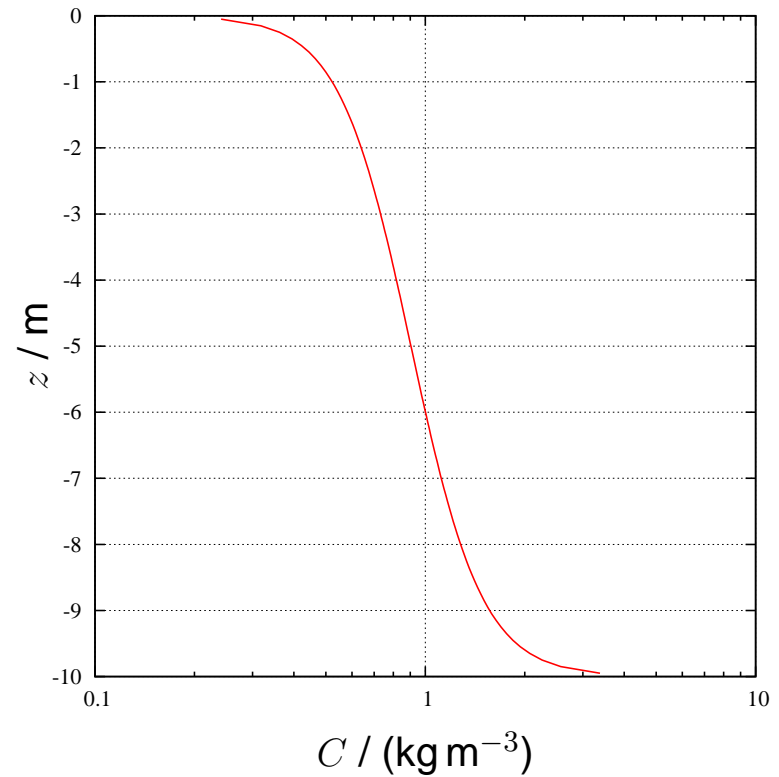
Rouse profile

Analytical Solution

Concentration: linear

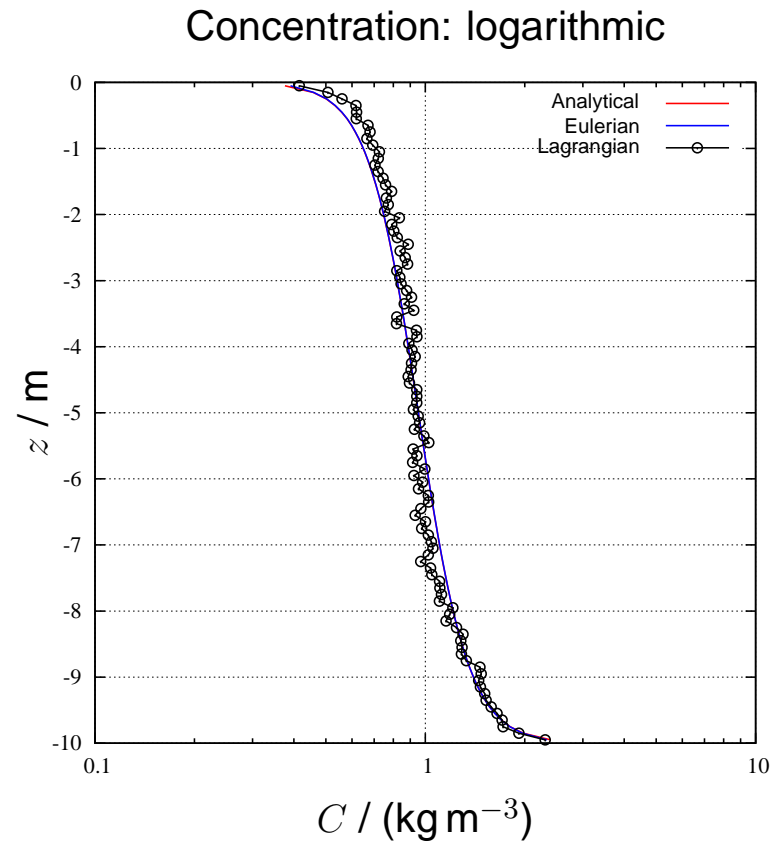
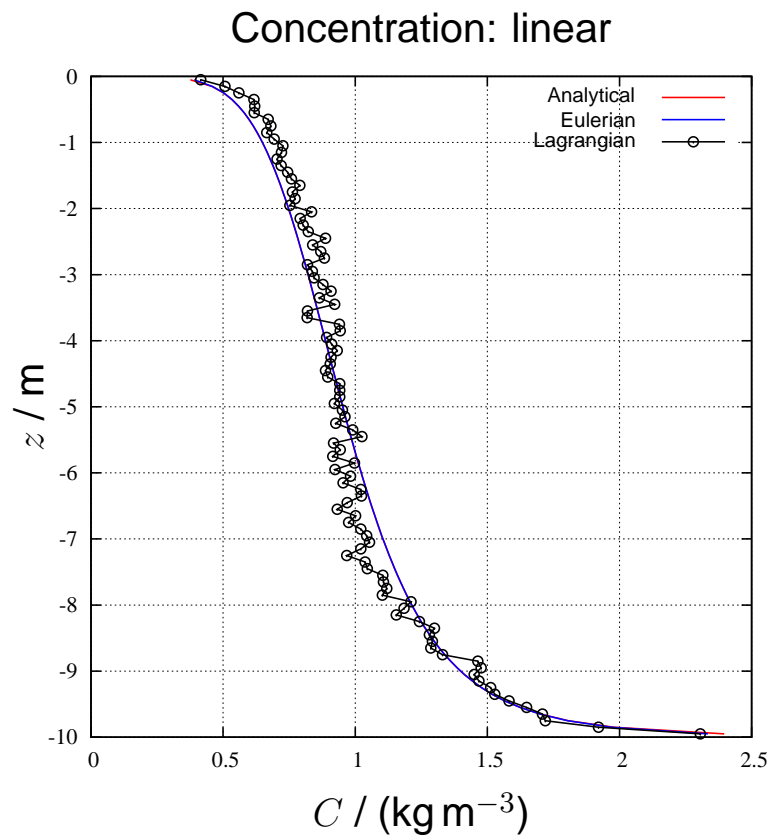


Concentration: logarithmic



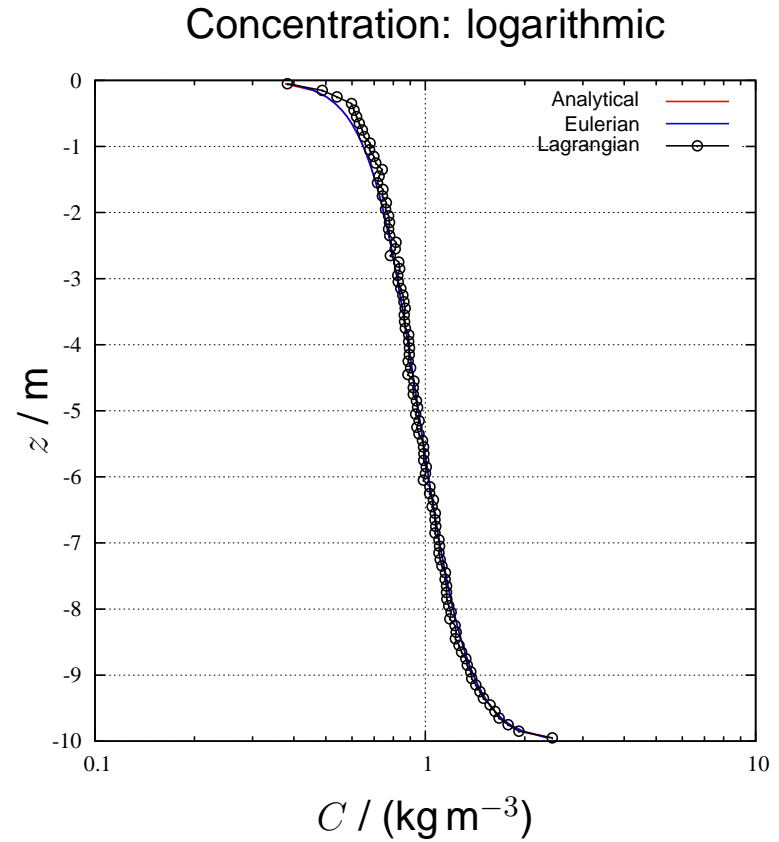
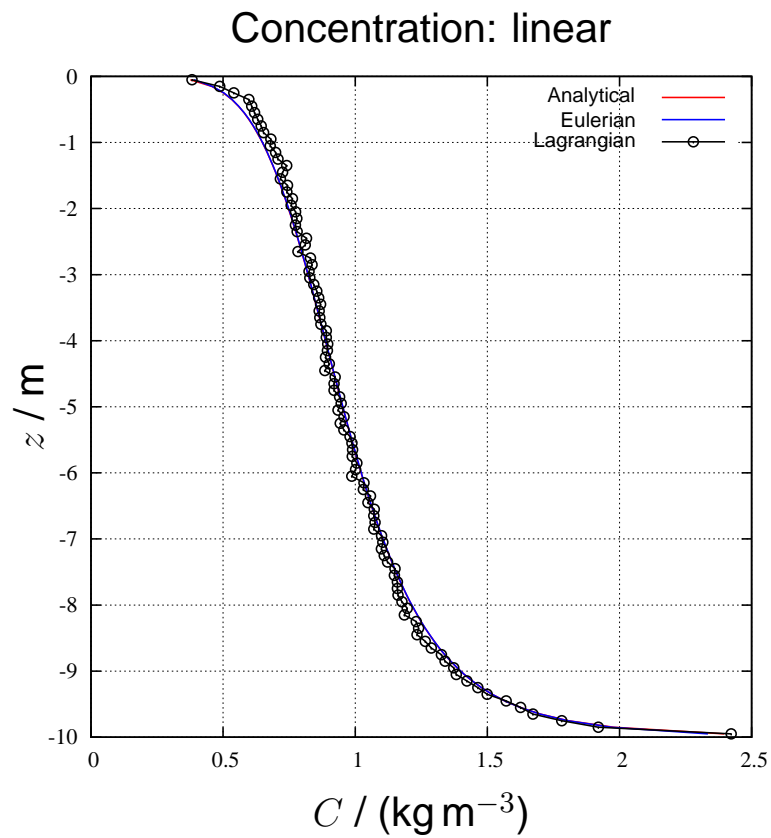
Rouse profile

Numerical Solution for $N = 1000$ Particles



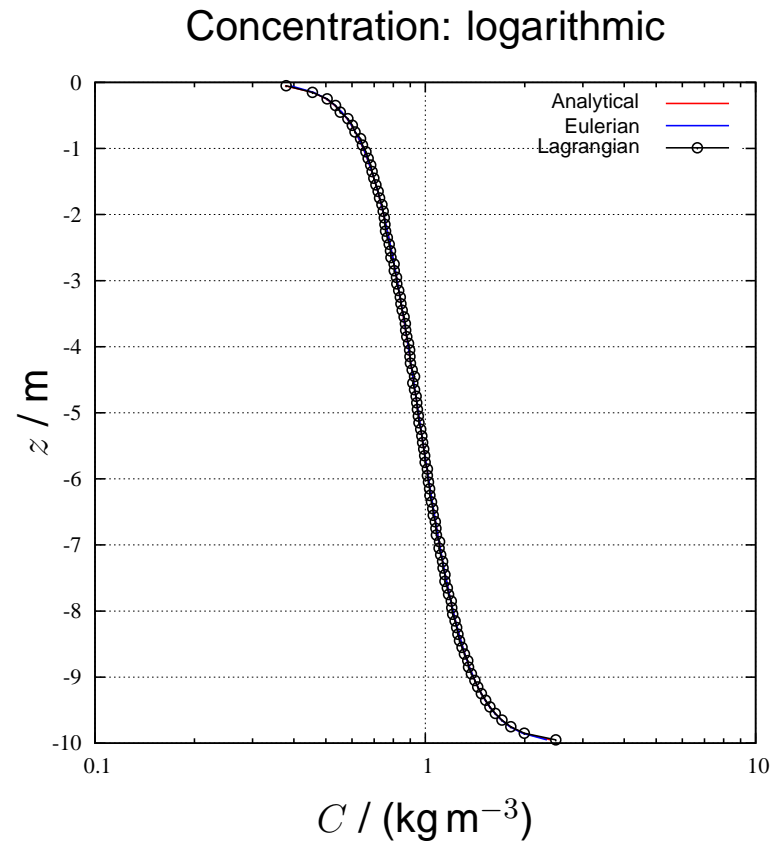
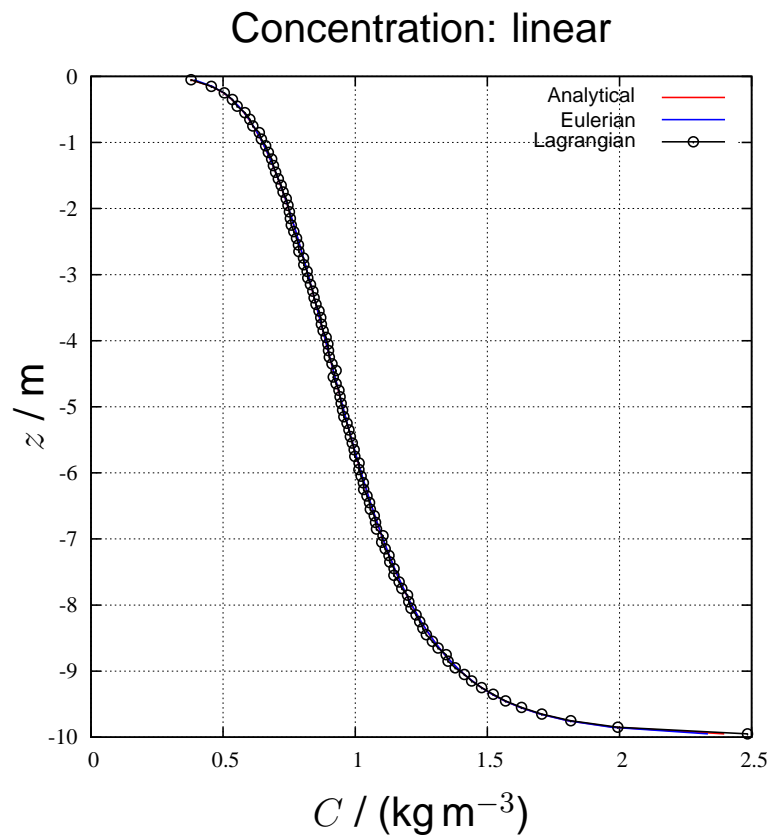
Rouse profile

Numerical Solution for $N = 10000$ Particles



Rouse profile

Numerical Solution for $N = 100000$ Particles



Rouse profile

Error with respect to analytical solution
(standard deviation)

Lagrangian scheme

| | | | |
|----------|--------|--------|--------|
| N | 10^3 | 10^4 | 10^5 |
| σ | 0.059 | 0.023 | 0.010 |

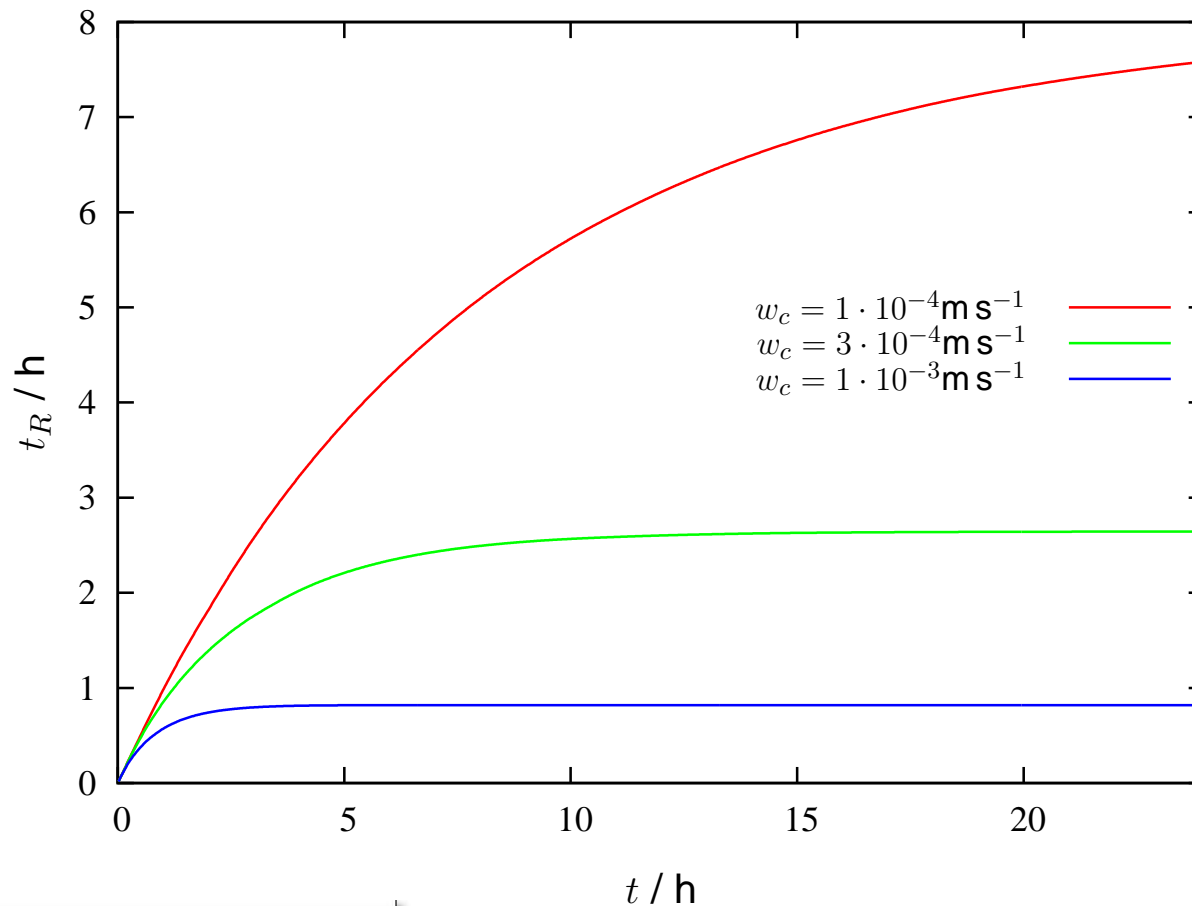
Eulerian scheme

| | | |
|------------------|--------|--------|
| Advection scheme | upwind | TVD |
| σ | 0.011 | 0.0064 |

Residence time

Residence time in water column (above $z = -9$ m)
Start at $z = 5$ m, $N = 10000$

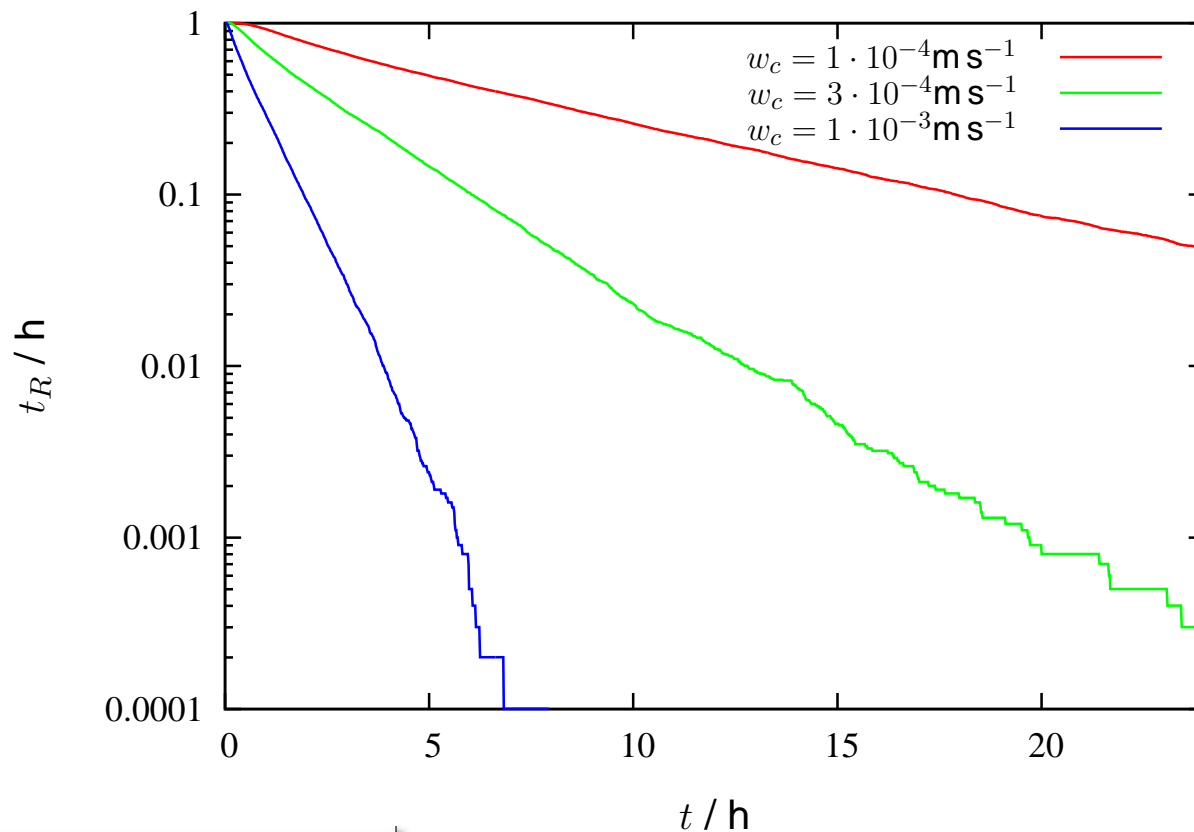
Evolution of Residence Time in h, Rouse no.=0.1



Residence time

Residence time in water column (above $z = -9$ m)
Start at $z = 5$ m, $N = 10000$

Fraction left in mixed layer, Rouse no.=0.1



Mussel filtration

Benthic filter feeders as sinks for suspended particulate matter

Eulerian formulation of boundary condition:

$$\nu_t \partial_z C \Big|_{z=-H} = w_f C \Big|_{z=-H}, \quad w_f = N_m V_f \quad (9)$$

w_f : **filtration velocity**

N_m : **number of mussels per m² (e.g. 1000)**

V_f : **filtration volume per mussel (e.g. 18 l per hour)**

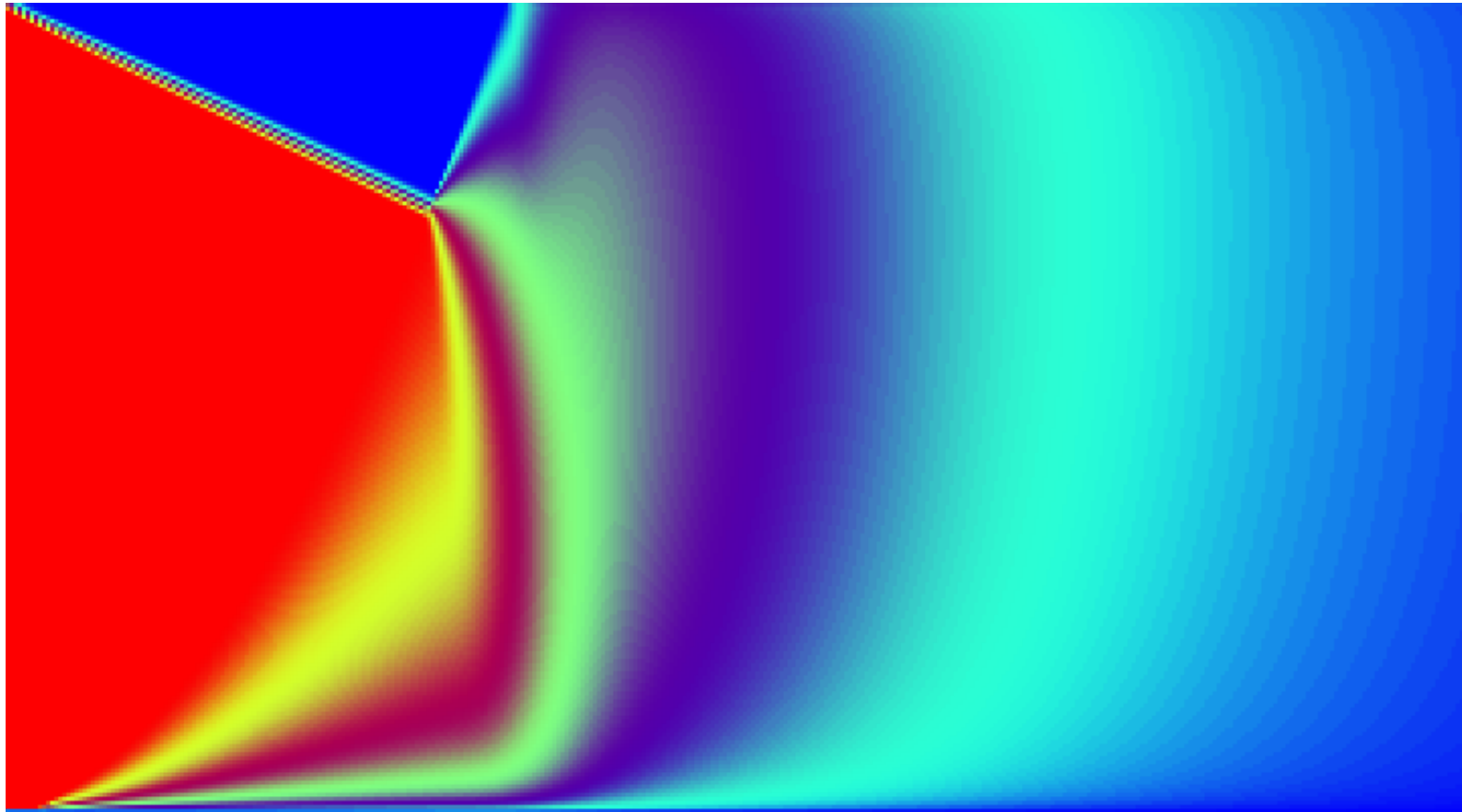
Lagrangian formulation of boundary condition:

At each time step take out all particles which are below

$$z = -H + \Delta t w_f.$$

Mussel filtration

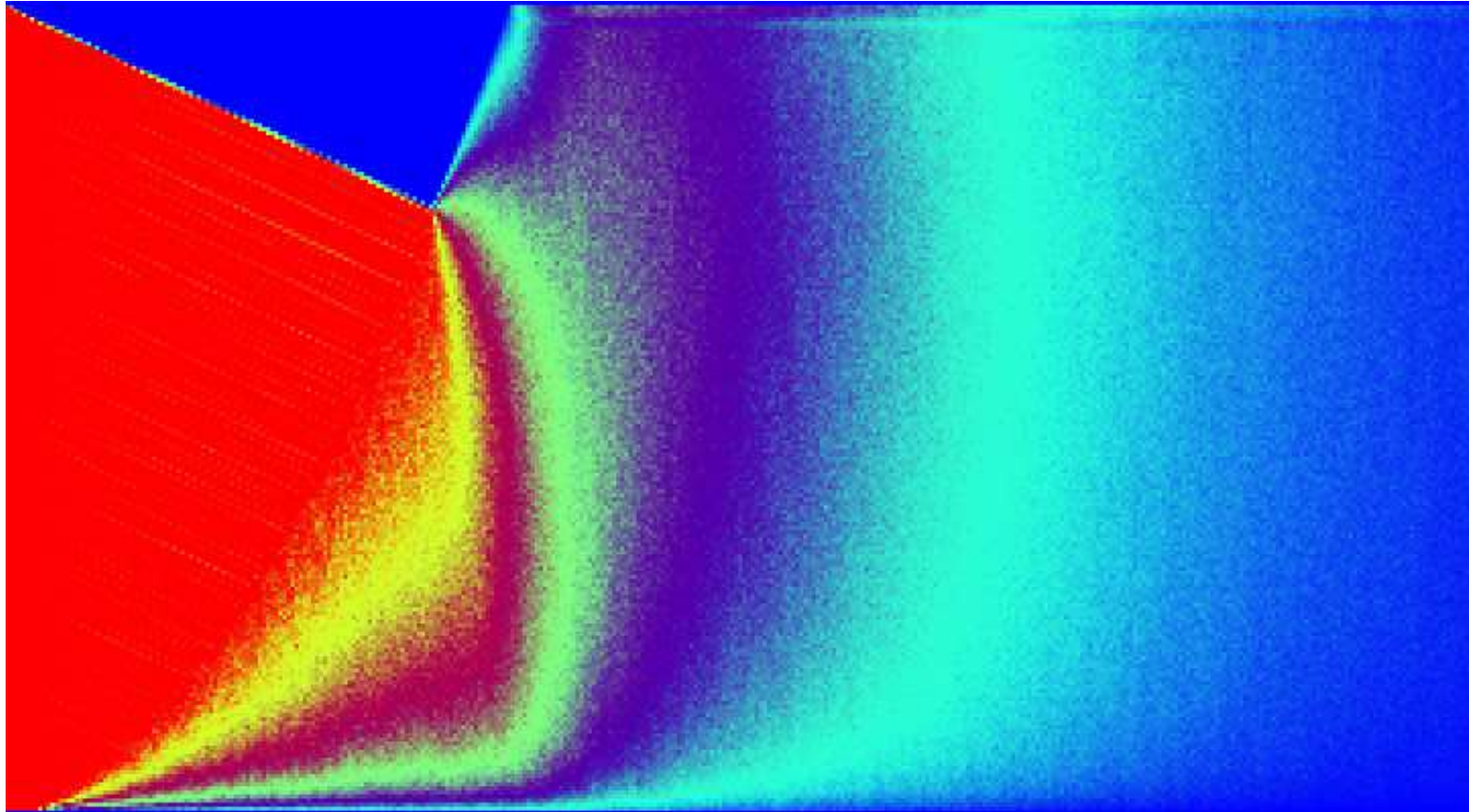
Profiles of suspended matter: Euler



(x -axis: $0\text{h} \leq t \leq 12\text{h}$, z -axis: $-10\text{m} \leq z \leq 0\text{m}$, contours: $0 \leq C \leq 1$)

Mussel filtration

Profiles of suspended matter: Lagrange

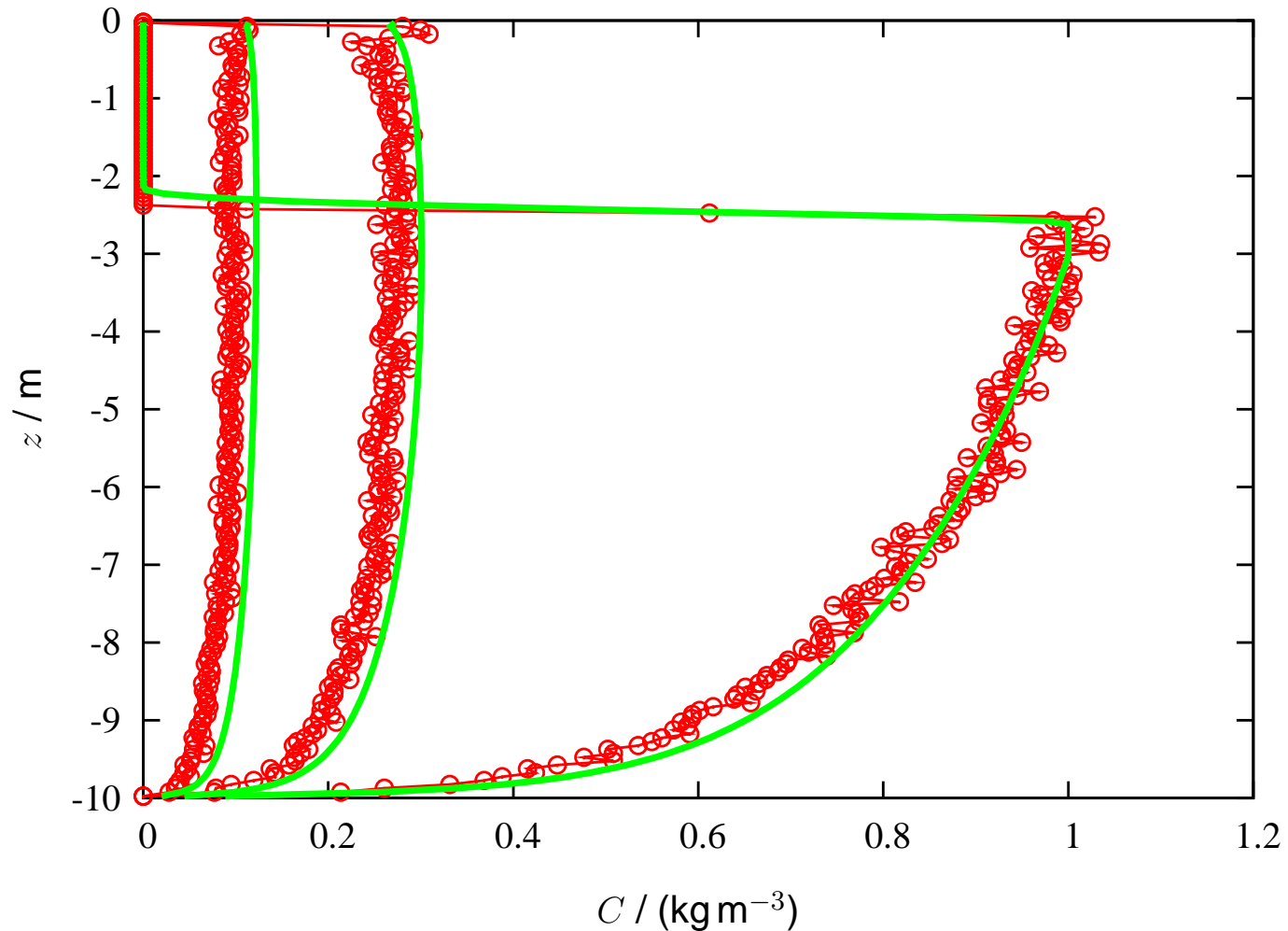


(x -axis: $0\text{h} \leq t \leq 12\text{h}$, z -axis: $-10\text{m} \leq z \leq 0\text{m}$, contours: $0 \leq C \leq 1$)

Mussel filtration

Profiles of suspended matter: Euler vs. Lagrange

Profiles at $t = 3 \frac{1}{3}; 6 \frac{2}{3}; 10$ h



Project: AlgaLag

A Lagrangian Study of Algal Bloom in the Ocean Mixed Layer (AlgaLag)

- German-Japanese Cooperation
- DFG-funding 2004-2007 (Travel expenses to Japan)
- Partner: Prof. Hidekatsu Yamazaki (Tokyo University of Marine Science and Technology)
- Objectives:
 - State-of-the-art Lagrangian model into GOTM
 - Improve random walk models for turbulence interaction
 - Lagrangian model for ecosystem models
 - Include memory effects into Lagrangian model
 - Improved parameterisations for Eulerian models

Photo adaptation

Let any Lagrangian particle represent a phytoplankton cell. Then the photosynthetic production rates may be calculated according to Nagai et al. [2003] as

$$P = P_d + Y(P_l - P_d) \quad (10)$$

with production rates for uninhibited and inhibited cells,

$$P_d = P_{dm} \left(1 - \exp \left(-\frac{PAR}{E_d} \right) \right), \quad (11)$$

$$P_l = P_{lm} \left(1 - \exp \left(-\frac{PAR}{E_l} \right) \right),$$

with P_{dm} , P_{lm} , E_d and E_l being constants.

Photo adaptation, cont'd

Y is the inhibition parameter defined by

$$\frac{dY}{dt} = \frac{1}{\gamma}(X - Y), \quad (12)$$

with the adaptation time scale γ and the instantaneous inhibition parameter

$$X = 1 - \exp \left(- \left(\frac{\max\{PAR, E_b\} - E_b}{E_b} \right)^2 \right). \quad (13)$$

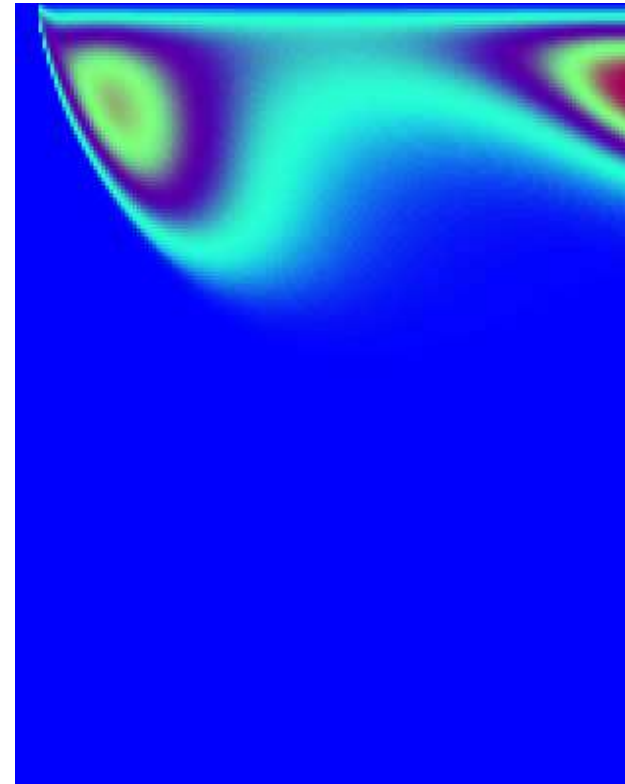
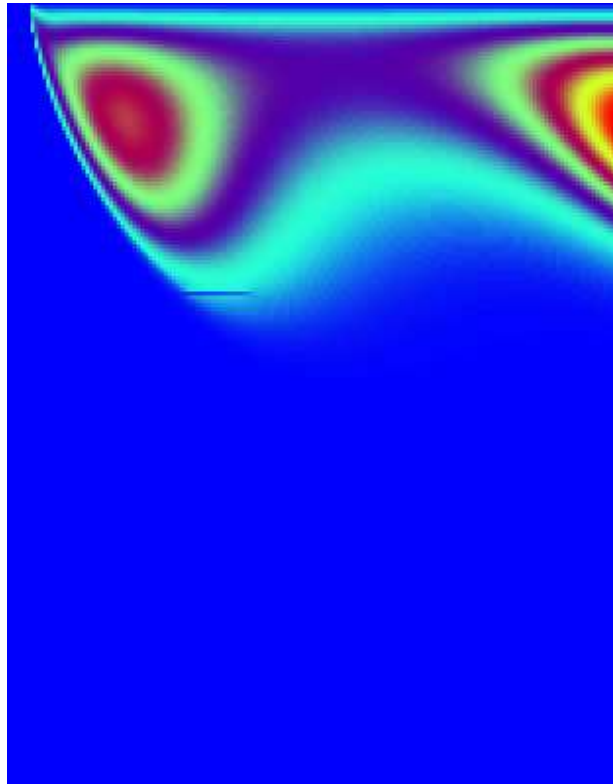
PAR is the photosynthetically available radiation.

Some experiments with variations in surface forcing ($W = 0, 5, 10 \text{ m s}^{-1}$) and turbidity (Jerlov I and III, coastal) have been carried out, see the following slides.

Photo adaptation

$W = 10 \text{ m s}^{-1}$, turbidity: Jerlov I (left), coastal (right)

Eddy diffusivity



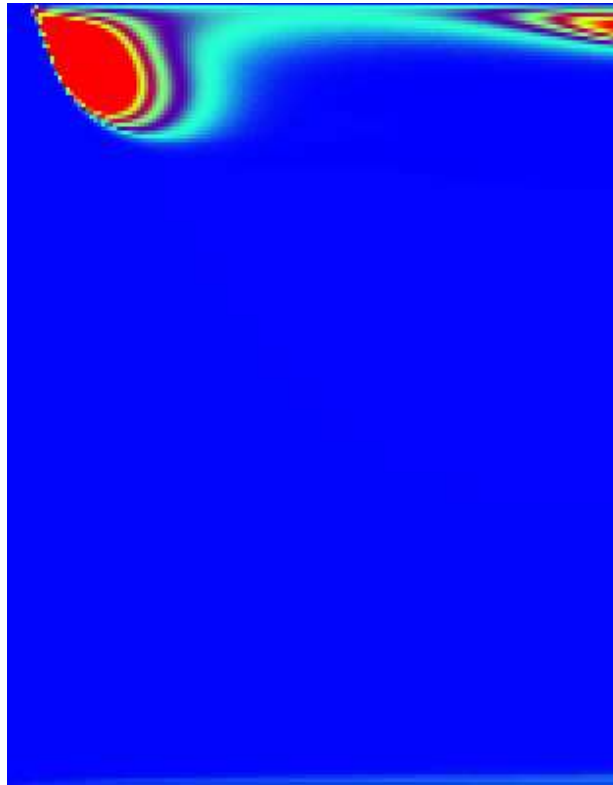
x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours: $0 \leq \nu'_t \leq 0.04 \text{ m}^2 \text{ s}^{-1}$

Photo adaptation

$W = 5 \text{ m s}^{-1}$, turbidity: Jerlov I (left), coastal (right)

Eddy diffusivity



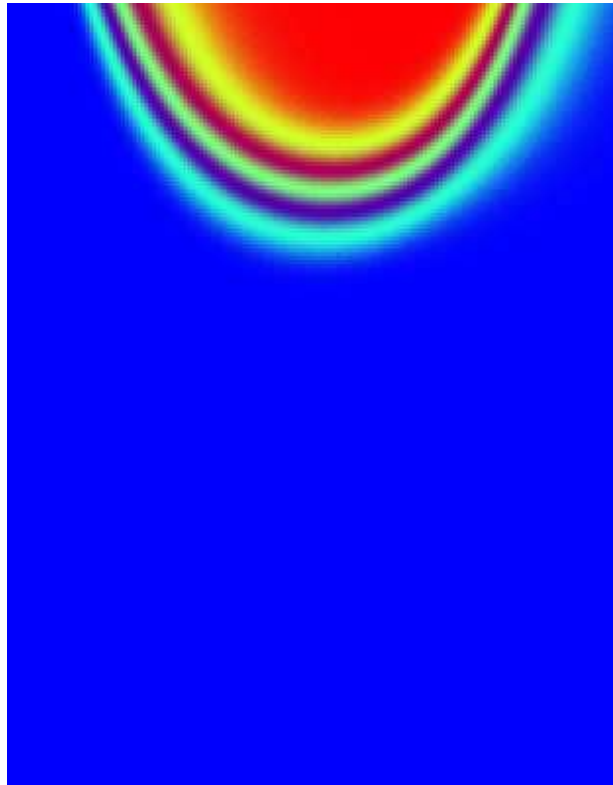
x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours: $0 \leq \nu'_t \leq 0.004 \text{ m}^2\text{s}^{-1}$

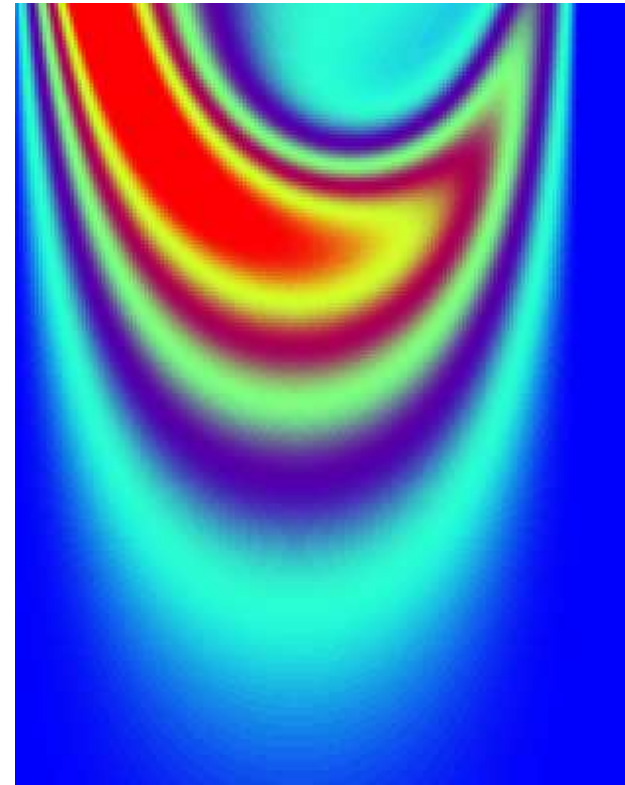
Photo adaptation

$W = 0 \text{ m s}^{-1}$, turbidity: Jerlov I

Inhibition parameter



Photosynthetic production rate



x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours left: $0 \leq Y \leq 1$; right: $0 \leq P \leq 0.004 \text{ pg-at O}_2 \text{ cell}^{-1} \text{ h}^{-1}$

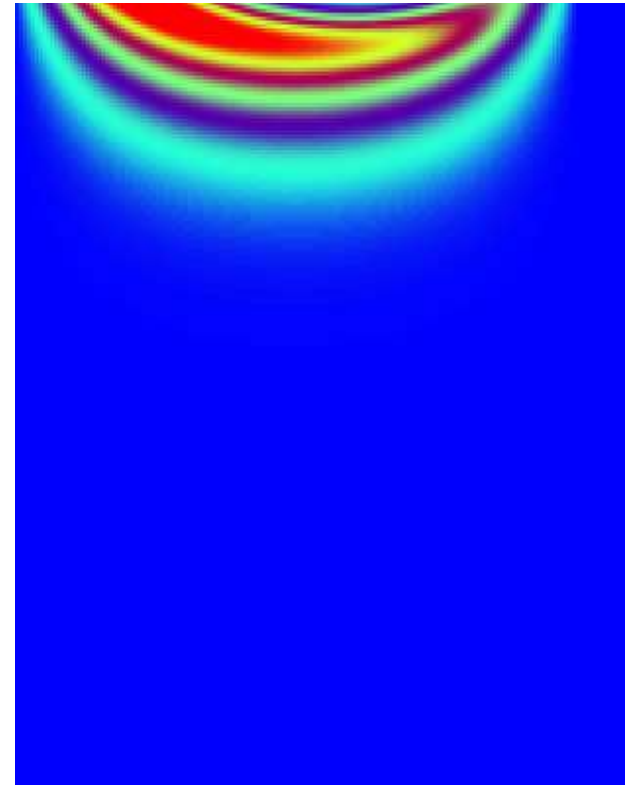
Photo adaptation

$W = 0 \text{ m s}^{-1}$, turbidity: Jerlov III

Inhibition parameter



Photosynthetic production rate



x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours left: $0 \leq Y \leq 1$; right: $0 \leq P \leq 0.004 \text{ pg-at O}_2 \text{ cell}^{-1} \text{ h}^{-1}$

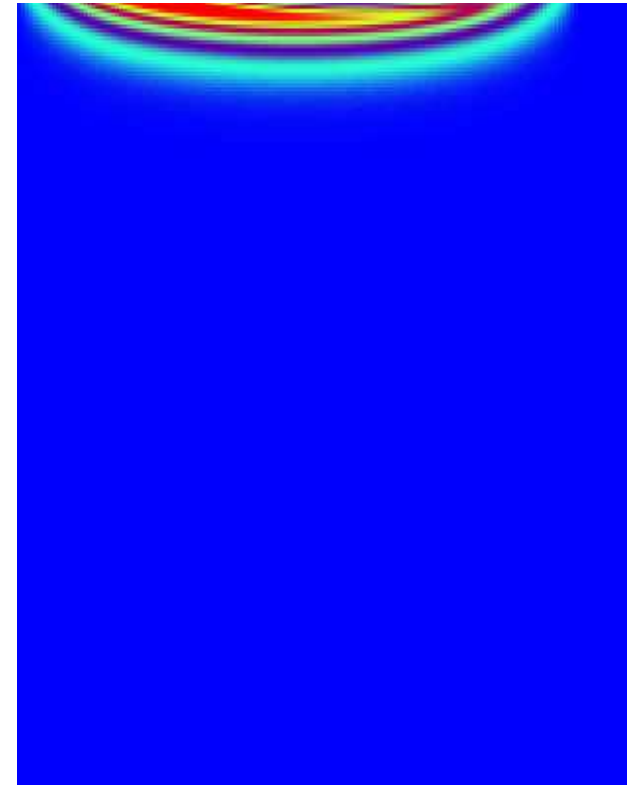
Photo adaptation

$W = 0 \text{ m s}^{-1}$, turbidity: coastal water

Inhibition parameter



Photosynthetic production rate



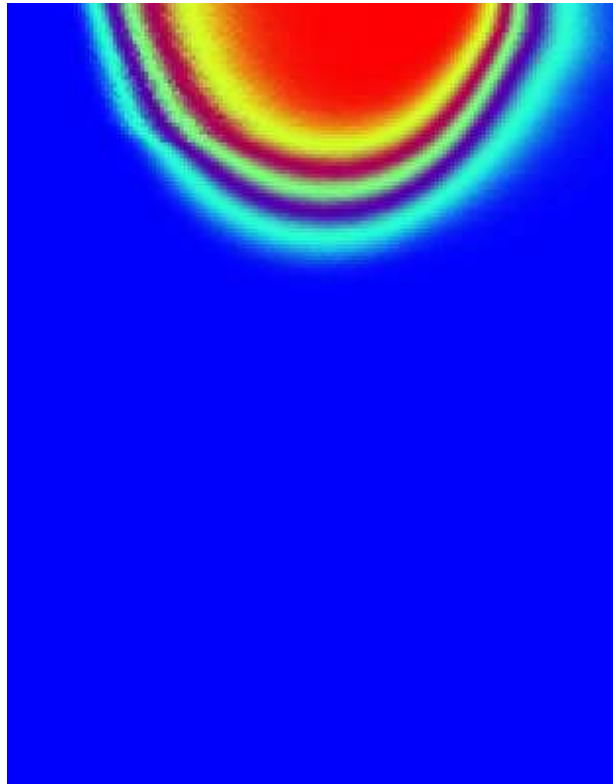
x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

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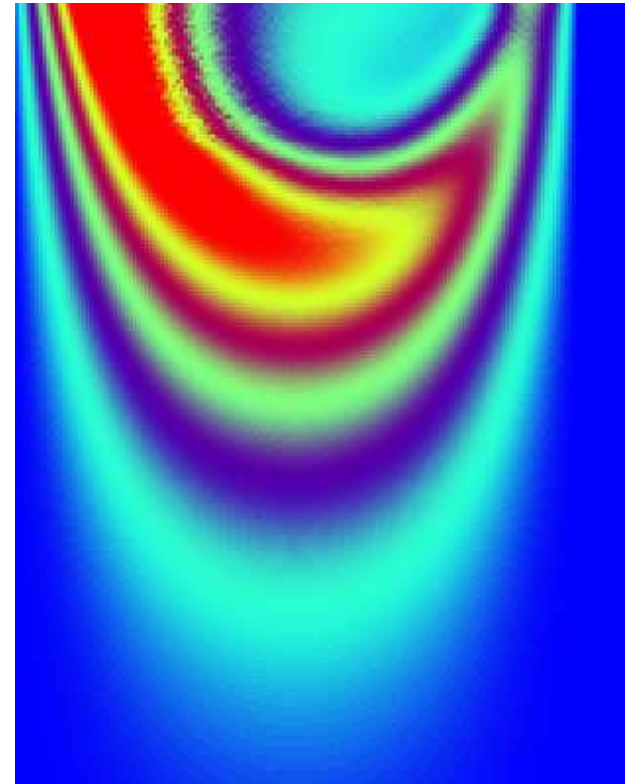
Photo adaptation

$W = 5 \text{ m s}^{-1}$, turbidity: Jerlov I

Inhibition parameter



Photosynthetic production rate



x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours left: $0 \leq Y \leq 1$; right: $0 \leq P \leq 0.004 \text{ pg-at O}_2 \text{ cell}^{-1} \text{ h}^{-1}$

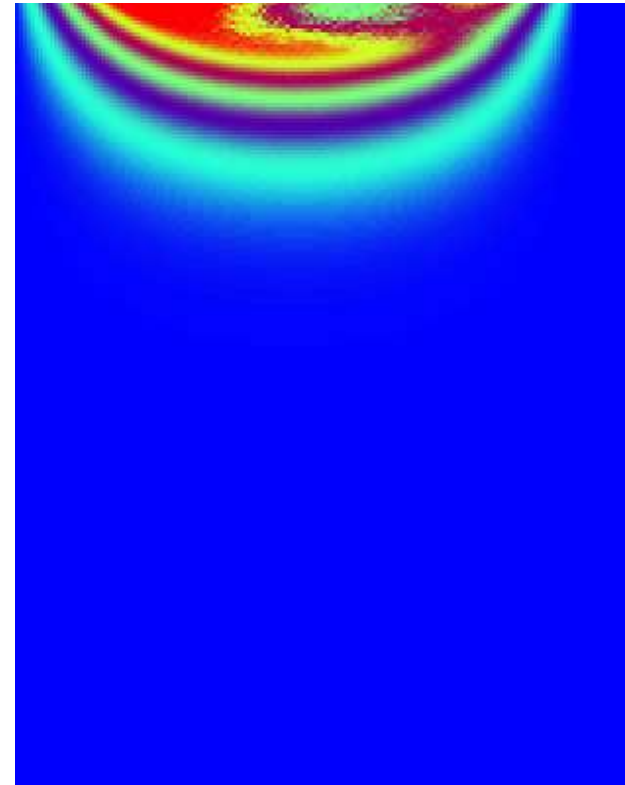
Photo adaptation

$W = 5 \text{ m s}^{-1}$, turbidity: Jerlov III

Inhibition parameter



Photosynthetic production rate



x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours left: $0 \leq Y \leq 1$; right: $0 \leq P \leq 0.004 \text{ pg-at O}_2 \text{ cell}^{-1} \text{ h}^{-1}$

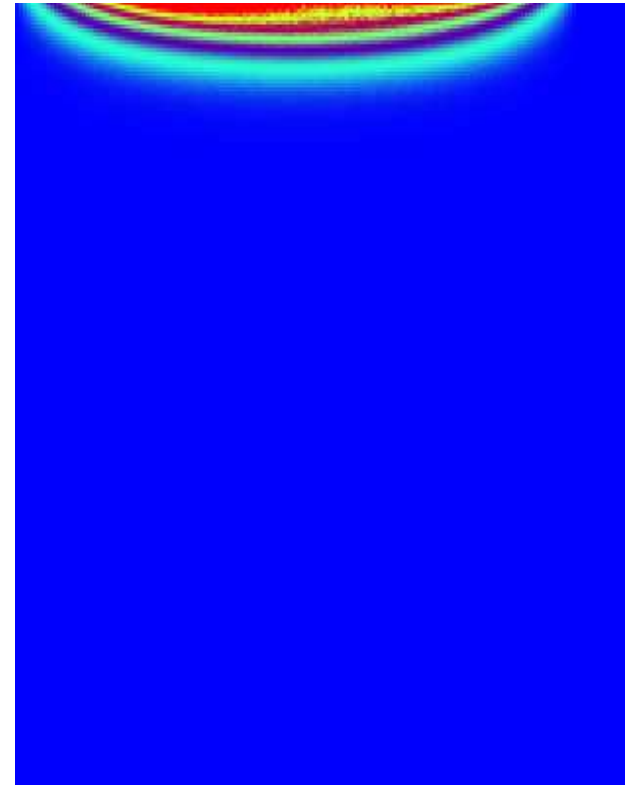
Photo adaptation

$W = 5 \text{ m s}^{-1}$, turbidity: coastal water

Inhibition parameter



Photosynthetic production rate



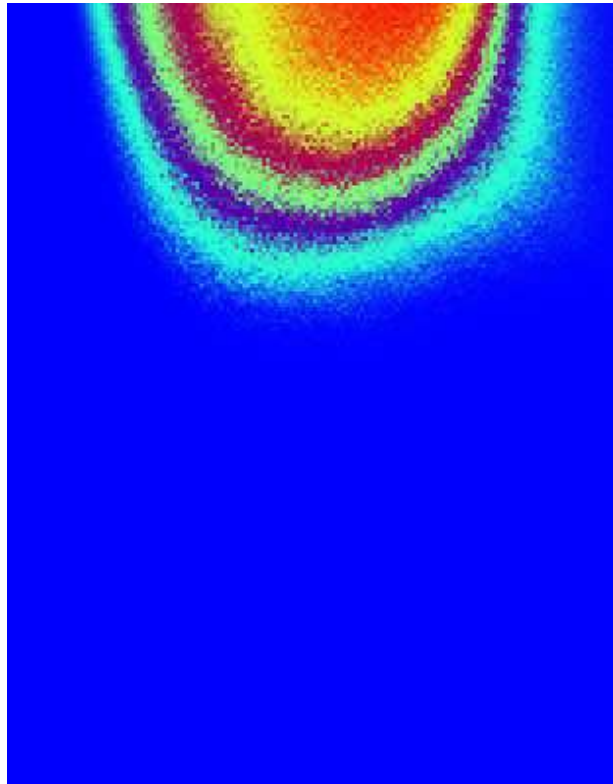
x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours left: $0 \leq Y \leq 1$; right: $0 \leq P \leq 0.004 \text{ pg-at O}_2 \text{ cell}^{-1} \text{ h}^{-1}$

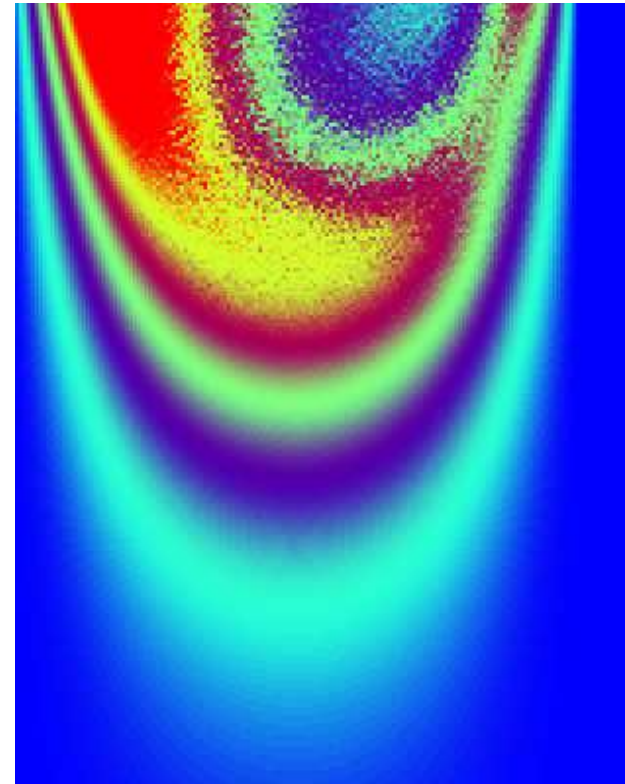
Photo adaptation

$W = 10 \text{ m s}^{-1}$, turbidity: Jerlov I

Inhibition parameter



Photosynthetic production rate



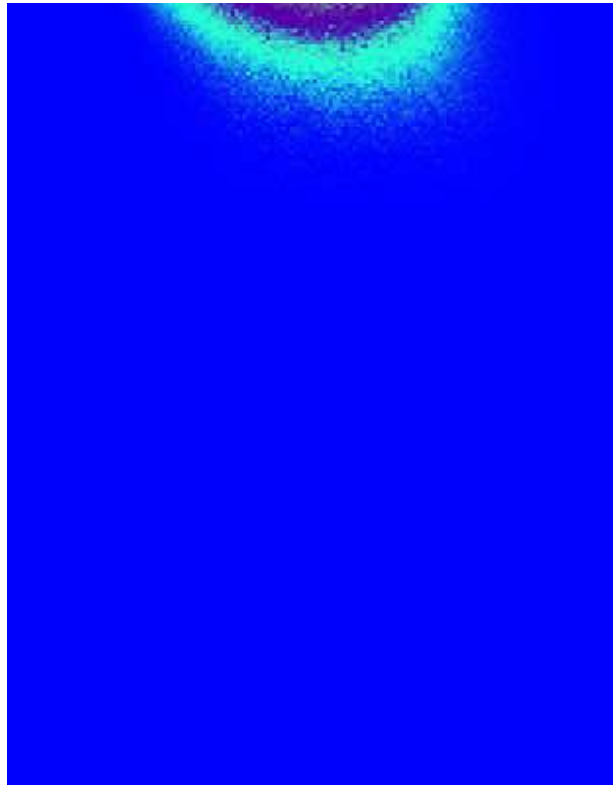
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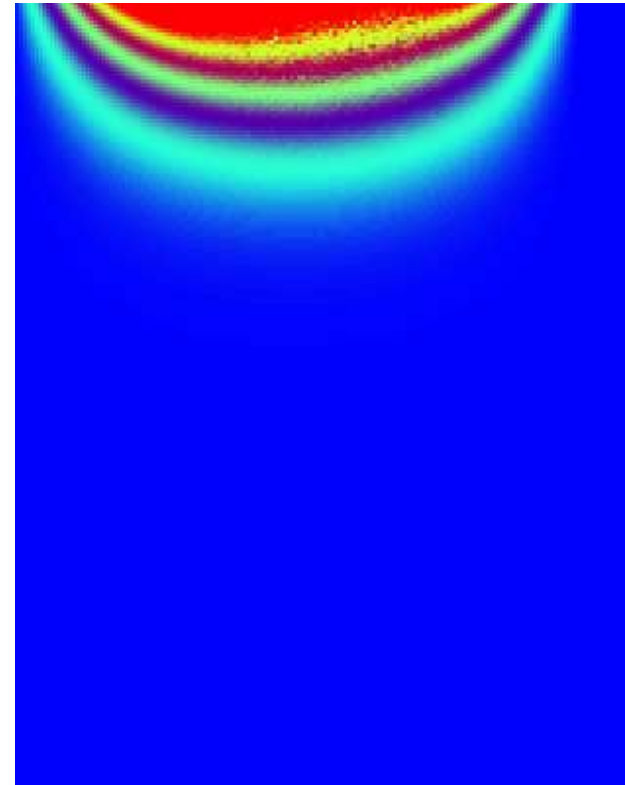
Photo adaptation

$W = 10 \text{ m s}^{-1}$, turbidity: Jerlov III

Inhibition parameter



Photosynthetic production rate



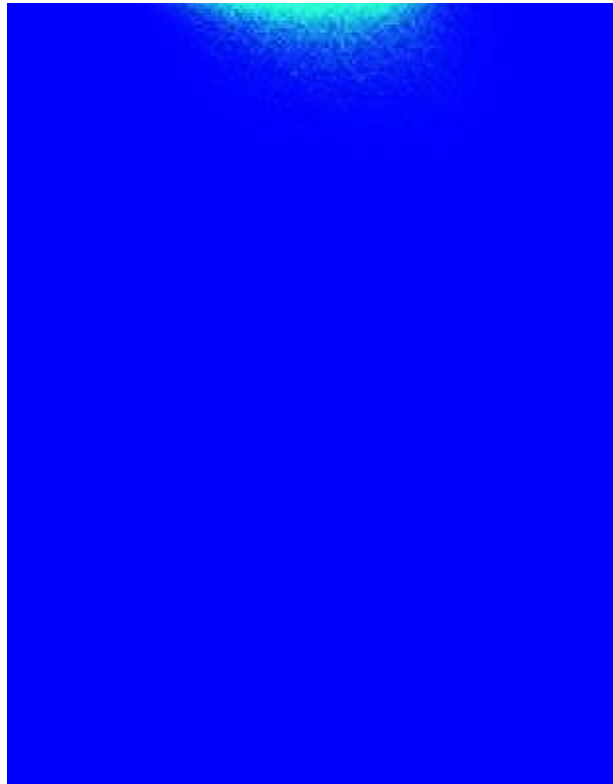
x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours left: $0 \leq Y \leq 1$; right: $0 \leq P \leq 0.004 \text{ pg-at O}_2 \text{ cell}^{-1} \text{ h}^{-1}$

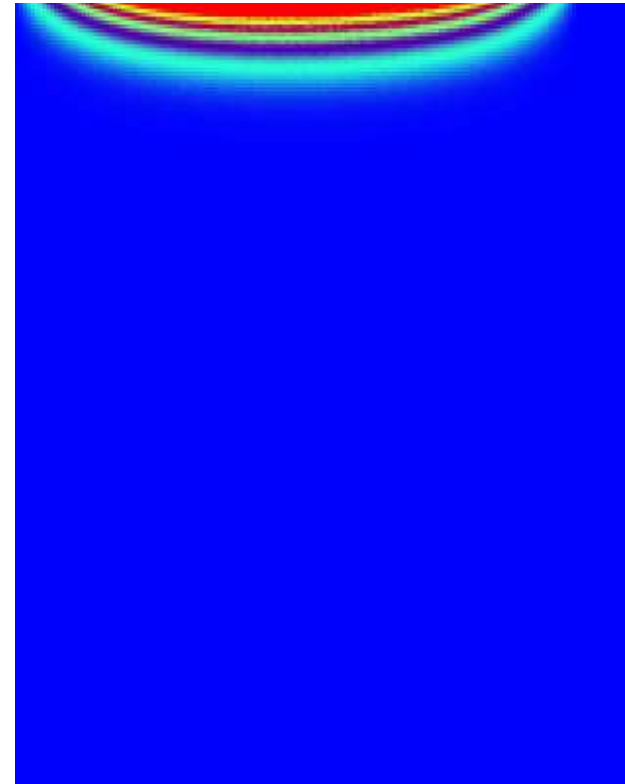
Photo adaptation

$W = 10 \text{ m s}^{-1}$, turbidity: coastal water

Inhibition parameter



Photosynthetic production rate



x -axis: $6:00 \text{ h} \leq t \leq 19:00 \text{ h}$, z -axis: $-100 \text{ m} \leq z \leq 0 \text{ m}$

Contours left: $0 \leq Y \leq 1$; right: $0 \leq P \leq 0.004 \text{ pg-at O}_2 \text{ cell}^{-1} \text{ h}^{-1}$