Assignment Nr. 2

due 11 May

Problem 1

Consider the Lorenz-equations,

$$\begin{aligned}\frac{\mathrm{d}X}{\mathrm{d}t} &= P_r(Y - X) , \\ \frac{\mathrm{d}Y}{\mathrm{d}t} &= -XZ + rX - Y , \\ \frac{\mathrm{d}Z}{\mathrm{d}t} &= XY - bZ ,\end{aligned} \tag{1}$$

with the constant coefficients $b = 8/3$, $Pr = 10$, and $r = 28$. A simple numerical discretization of (1) is given by

$$\begin{aligned}\frac{X^{n+1} - X^n}{\Delta t} &= P_r(Y^n - X^n) , \\ \frac{Y^{n+1} - Y^n}{\Delta t} &= -X^n Z^n + rX^n - Y^n , \\ \frac{Z^{n+1} - Z^n}{\Delta t} &= X^n Y^n - bZ^n ,\end{aligned} \tag{2}$$

where the super-script n denotes the 'old' values, $n+1$ the 'new' values, and Δt the size of the time step.

(a) Describe (in words) the physics contained in (1).

(b) Design a computer program solving (2) for arbitrary inital conditions, time steps, and integration times. The source code has to be delivered.

(c) Sensitivity to inital conditions: Integrate the model for $0 \leq t \leq 100$, using a time step of $\Delta t = 0.001$, and initialize the model with $(X_0, Y_0, Z_0) = (15, 15, 15)$ and $(X_0, Y_0, Z_0) = (15, 15, 15.00001)$. Plot and discuss the results.

(d) Stability of attractor point: Integrate the model for $0 \leq t \leq 100$, using a time step of $\Delta t = 0.001$, and initialize the model with an attractor point, and a point which is located 0.00001 units away from it in every coordinate direction. Plot and discuss the results.

(e) Sensitivity to numerical discretization: Integrate the model for $0 \leq t \leq 100$, using the time steps $\Delta t = 0.001$ and $\Delta t = 0.00100001$, respectively, and initialize the model with $(X_0, Y_0, Z_0) = (15, 15, 15)$. Plot and discuss the results.